Scalar-Tensor theories and Renormalization Group effects in gravity

FAPES
FUNDAÇÃO DE AMPARO À PESQUISA DO ESPÍRITO SANTO

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

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VI Challenges of New Physics in Space

Campos do Jordão-SP, Brazil
May 26, 2015

Introduction

It is well known that coupling constants effectively run depending the the energy scale of the experiment.

For electromagnetism, the interactions become stronger at higher energies (smaller distances). In particular, the fine structure constant at low energies is ~ 1/137, while at 80 GeV it becomes ~1/128.

This behaviour is governed by the Renormalization Group (RG) equations. For electromagnetism, in the limit of low energies, the effective charge behaves as a true constant (the beta function goes to zero).

Introduction

In the case of gravity, namely in QFT in curved spacetime or certain quantum gravity approaches, the RG can be applied, but (contrary to QED or QCD for instance) there is no proof that G and Lambda indeed become constants at low energies *(the couplings of higher derivative terms do become constants)* [e.g., Gorbar, Shapiro, JHEP (2003), Shapiro, Solà, PLB (2009)].

Diverse works consider possible RG effects that extend General Relativity at large scales (with running of G and Lambda) [e.g., Reuter, Wetterich, PLB (1987), Goldman et al, PLB (1992), Reuter, Weyer PRD (2004), Shapiro, Sola, Stefancic JCAP (2005), Bauer CQG (2005), Brownstein, Moffat ApJ (2006), Borges, Carneiro, Fabris, Pigozzo PRD (2008), DCR, Shapiro, Letelier, JCAP (2010), Koch, Ramirez CQG (2011), Perico, Lima, Basilakos, Sola, PRD (2013)…]

In [DCR, B.Chauvineau, O.Piattella, 1504.05119] we tackled the following question:

At large scales, can RG extended gravity be expressed as a particular case of Scalar-Tensor (ST) gravity?

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Answer:

It can be expressed as a kind of ST gravity, but a *nonconventional* one, since:

a) simple cases from the RG perspective can lead to intricate scalar potentials that can easily be non-analytic expressions.

b) the corresponding potential is system-dependent, i.e. it may depend on constants that are associated with the matter description.

c) On looking for perturbative solutions, a natural scheme within the RG approach leads to an unusual perturbative scheme from the ST perspective.

Renormalization Group and gravity: principles

General Relativity is a perturbatively non-renormalizable theory. At sufficiently high energies, it is expected to yield wrong predictions.

A particular type of quantum correction, has been attracting interest currently, that of *nontrivial Renormalization Group (RG) flows*.

i) *High energy*: nonperturbative renormalizability with unitarity may be achieved from the Asymptotic Safety program [for a review, Niedermaier, Reuter, Living Rev. Rel. (2006)]).

ii) *Low energy*: ii a) Some RG flows that may work for the high energy case can be extended to low energy with nontrivial consequences, effectively modifying classical General Relativity at large scales [e.g., Reuter, Weyer, JCAP (2004)].

ii b) Independently on the Asymptotic Safety program, the beta-functions of G and Lambda need not to satisfy the decoupling theorem [Appelquist, Carazzone, PRD, 2856 (1975)], hence they need not to quickly approach zero at low energies (large scales) [e.g., Tsamis, Woodard, Annals.Phys.(1995), Gorbar, Shapiro, JHEP (2003), Shapiro, Solà, PLB (2009)].

RG and gravity: Three implementation types

At what level should the constant G be improved to $G(\mu)$?

Classification from [Reuter, Weyer PRD 2004]:

➡ *RG improved solutions*:
After the classical solutions are derived.

➡ *RG improved equations*:
After the field equations are evaluated from the action. Example:

$$G_{\alpha\beta} + 2\Lambda(\mu) = 8\pi G(\mu) T_{\alpha\beta}$$

➡ *RG improved action*:
At the action level. Example:

$$S = \frac{1}{16\pi} \int \frac{R - 2\Lambda(\mu)}{G(\mu)} \sqrt{-g} d^4x$$

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External field formulation: the action

In [Reuter, Weyer, PRD 2004; Shapiro, Solà, Stefancic, JCAP 2005] it is argued in favour of the following simplest action capable of enclosing the RG effects for gravity at large scales,

$$S[g, \Psi] = \frac{1}{16\pi} \int \frac{R - 2\Lambda}{G} \sqrt{-g} d^4x + S_{\mathrm{m}}[g, \Psi]$$

External fields. Obtained from semi-classical/quantum considerations

It is natural that G and Lambda should behave as *external fields* from the perspective of a classical action.

There is a single field eq. from the above action, namely

$$\underbrace{G_{\alpha\beta} + G\Box G^{-1} g_{\alpha\beta} - G\nabla_\alpha \nabla_\beta G^{-1}}_{\equiv \mathcal{G}_{\alpha\beta}} + \Lambda g_{\alpha\beta} = 8\pi G T_{\alpha\beta},$$

External field formulation: Beta-function and scale setting

The previous equation can only fix the dynamics in case additional information on G and Lambda are provided

To this end, two considerations from the RG perspective are relevant:
i) the beta-functions;
ii) the meaning of the RG scale (μ).

i) For instance, providing a beta function for G implies the knowledge of G as a function of a scale μ , i.e.,

$$\beta_G \Rightarrow G(\mu)$$

ii) Fixing the meaning of the scale μ should fix its relation to other physical quantities, hence it should provide a function f such that

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$$\mu = f(g, \gamma, \Psi)$$

Spacetime metric

Matter fields of any nature

Possible background geometry dependence (e.g. Minkowski)

External field formulation: the consistency equation

$$\nabla_\alpha T^{\alpha\beta} = 0 \implies \nabla_\alpha \left(\frac{\Lambda}{G} \right) = \frac{1}{2} R \nabla_\alpha G^{-1}$$

For a given metric, this imposes a relation between $G(\mu), \Lambda(\mu)$ and $\mu(x)$.

The external character of G and Lambda was pointed as the main difference between this RG approach and ST gravity [Reuter, Weyer, PRD 2004].

Can this approach be reformulated without external fields?

A formulation without external fields

It was realised [Koch, Ramirez CQG 2011, Hindmarsh, Saltas PRD 2012, Domazet, Stefancic CQG 2012] that the following action can generate field equations of the same form but without external fields,

$$S[g,\mu,\Psi] = \frac{1}{16\pi}\int \frac{R-2\Lambda(\mu)}{G(\mu)}\sqrt{-g}\, d^4x + S_{\rm m}[\Psi,g]$$

$$\frac{\delta S}{\delta g^{\alpha\beta}} \qquad\qquad \frac{\delta S}{\delta \mu}$$

$$\mathcal{G}_{\alpha\beta} + \Lambda g_{\alpha\beta} = 8\pi G T_{\alpha\beta}, \qquad\qquad \frac{d}{d\mu}\left(\frac{\Lambda}{G}\right) = \frac{1}{2}R\frac{d}{d\mu}G^{-1}.$$

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If $\nabla_\alpha \mu \neq 0.$

Otherwise, see [B.Chauvineau, DCR, J.Fabris, 1503.07581]

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Field equations with the same form of the external field approach!

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Derived scale setting (!!?)

$$\mu = F(R)$$

(at least locally).

If $\nabla_\alpha \mu \neq 0$.

Otherwise, see [B.Chauvineau, DCR, J.Fabris, 1503.07581]

Field equations with the same form of the external field approach!

$$\nabla_\alpha\left(\frac{\Lambda}{G}\right) = \frac{1}{2} R \nabla_\alpha G^{-1}$$

Previous formulation without external fields: really equivalent?

In general they are not. [DCR, B.Chauvineau, O.Piattella, 1504.05119]

Within certain approaches [e.g., Reuter, Weyer, JCAP, PRD (2004); DCR, Shapiro, Letelier JCAP (2010); DCR, Oliveira, Fabris, Gentile MNRAS (2014)…], μ is directly fixed from semi-classical arguments, while the consistency equation imposes a restriction on the beta functions for G and Lambda.

We will consider the following framework:

i) RG improved action

ii) Fixed $G(\mu)$ at the action level (equivalently, fixed β_G)

iii) Scale setting at the action level ($\mu = f(g, \gamma, \Psi)$)

iv) The relation between $\Lambda \text{ and } \mu$ is derived from the field equations.

An effective action for RG extended General Relativity

In order to generate the mentioned properties, we propose the following effective action

$$S[g,\gamma,\mu,\lambda,\Psi] = \int \left[\frac{R - 2\Lambda\{\mu\}}{16\pi G(\mu)} + \lambda\left(\mu - f(g,\gamma,\Psi)\right) \right] \sqrt{-g}\, d^4x + S_{\mathrm{m}}[g,\Psi].$$

Contrary to previous approaches, the above action contains *all* the information necessary to evaluate the dynamics.

Equivalently, one can eliminate the “constraint" and write,

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Denotes a scalar that depends on spacetime from μ , but the dependence form will be derived from the field equations.

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A covariant scale setting framework within a relativistic fluid

Apart from the Ricci scalar case, almost all the scale settings used in the literature specify it in a given coordinate system. Their extension to covariant expressions need not to be trivial.

The following scale setting is both in accordance with the previous framework, and extends the our previous noncovariant proposal,

$$\mu = f(\Phi_N) \longrightarrow \mu = f\left(U^\alpha U^\beta h_{\alpha\beta}\right),$$

where U^α is the quadrivelocity of the fluid, $h_{\alpha\beta} \equiv g_{\alpha\beta} - \gamma_{\alpha\beta}$,
$\gamma_{\alpha\beta}$ is a background with respect to the perturbation h.

And we use the following expression already derived from diverse approaches [e.g., Reuter, Weyer PRD 2004, Shapiro, Solà, Stefancic JCAP 2005, Bauer CQG 2005],

$$G(\mu) = \frac{G_0}{1 + 2\nu \ln \mu}$$

ν is a dimensionless small parameter.

$\nu = 0$ corresponds to standard GR.

There are some actions that describe a relativistic fluid. We consider the formulation of [Ray, J.Math.Phys. 1972],

$$S_{\mathrm{f}}[g, U, n, s, \eta_p, X] = \int \left[-\rho(n,s) + \eta_1 (1 + U^\alpha U_\alpha) + \eta_2 \nabla_\alpha (n U^\alpha) + \eta_3 U^\alpha \nabla_\alpha X + \eta_4 U^\alpha \nabla_\alpha s\right] \sqrt{-g}\, d^4x.$$

rest mass density

specific entropy

Langrange multipliers

particle identity

4-velocity constraint

mass conservation

particle id conservation (important for rotational flow)

particle entropy conservation

$$S[g,\mu,\lambda,U,\gamma,...] = \int \left[\frac{R-2\Lambda\{\mu\}}{16\pi G(\mu)} + \lambda\left(\mu - f(U^\alpha U^\beta h_{\alpha\beta})\right) \right] \sqrt{-g}\, d^4x + S_{\rm f}[g,U,...].$$

$$S_{\rm f}[g,U,n,s,\eta_p,X] = \int \left[-\rho(n,s) + \eta_1(1+U^\alpha U_\alpha) + \eta_2 \nabla_\alpha(nU^\alpha) + \eta_3 U^\alpha \nabla_\alpha X + \eta_4 U^\alpha \nabla_\alpha s\right] \sqrt{-g}\, d^4x.$$

Considering all the field equations apart from the one from gamma,

$$\mathcal{G}_{\alpha\beta} + \Lambda g_{\alpha\beta} = 8\pi G \left[(P + \rho - 2(U^\kappa U^\sigma h_{\kappa\sigma} + 1)\lambda f')U_\alpha U_\beta + P g_{\alpha\beta} \right]$$

Single term that may modify the perfect fluid.

But,

$$\frac{\delta S}{\delta \gamma_{\alpha\beta}} = 0 \implies \lambda f' U^\alpha U^\beta = 0 \implies \lambda = 0$$

Conclusion: the use of a reference geometry given by $\gamma_{\alpha\beta}$ is important to guarantee $\nabla^\alpha T_{\alpha\beta} = 0$.

First order perturbative approach

Consider a first order perturbation on ν,

$$G^{-1}(\mu) = \frac{1 + 2\nu \ln \mu}{G_0}$$

$$g_{\alpha\beta} = \overset{(0)}{g}_{\alpha\beta} + \nu \overset{(1)}{g}_{\alpha\beta} + O(\nu^2),$$

$$T_{\alpha\beta} = \overset{(0)}{T}_{\alpha\beta} + \nu \overset{(1)}{T}_{\alpha\beta} + O(\nu^2),$$

$$\Lambda = \Lambda_0 + \nu\Lambda_1 + O(\nu^2)$$

First order perturbative approach: solution about de Sitter

The consistency equation can also be written as,

$$\nabla_\alpha \Lambda = \left(\frac{3}{2} G \Box G^{-1} + \Lambda - 4\pi G T \right) G \nabla_\alpha G^{-1}.$$

which, up to the first order on ν reads,

$$\nu \nabla_\alpha \Lambda_1 = \left(\Lambda_0 - 4\pi G_0 \overset{(0)}{T} \right) G_0 \nabla_\alpha G^{-1} + O(\nu^2).$$

From the above, the Lambda solution is seen to depend on the matter content.

If the zeroth order perturbation has no matter (or T = 0),

$$\boxed{\Lambda = \Lambda_0 G_0 G^{-1} + O(\nu^2) = \Lambda_0 (1 + 2\nu \ln \mu) + O(\nu^2)}$$

Perturbative solution about a constant density "star"

The exact GR solution for a constant density star of radius R and mass M is well known,

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\Omega^2,$$

with

$$A(r) = \left(\frac{3}{2}\sqrt{1 - 2\frac{M}{R}} - \frac{1}{2}\sqrt{1 - 2\frac{M}{R^3}r^2} \right)^2,$$

$$B(r) = \frac{1}{1 - 2\frac{M}{R^3}r^2},$$

In order to evaluate the interior solution r < R, it is necessary to specify a particular f function for the scale setting. As a toy model, whose solution is qualitative interesting and not as lengthy than other options, consider

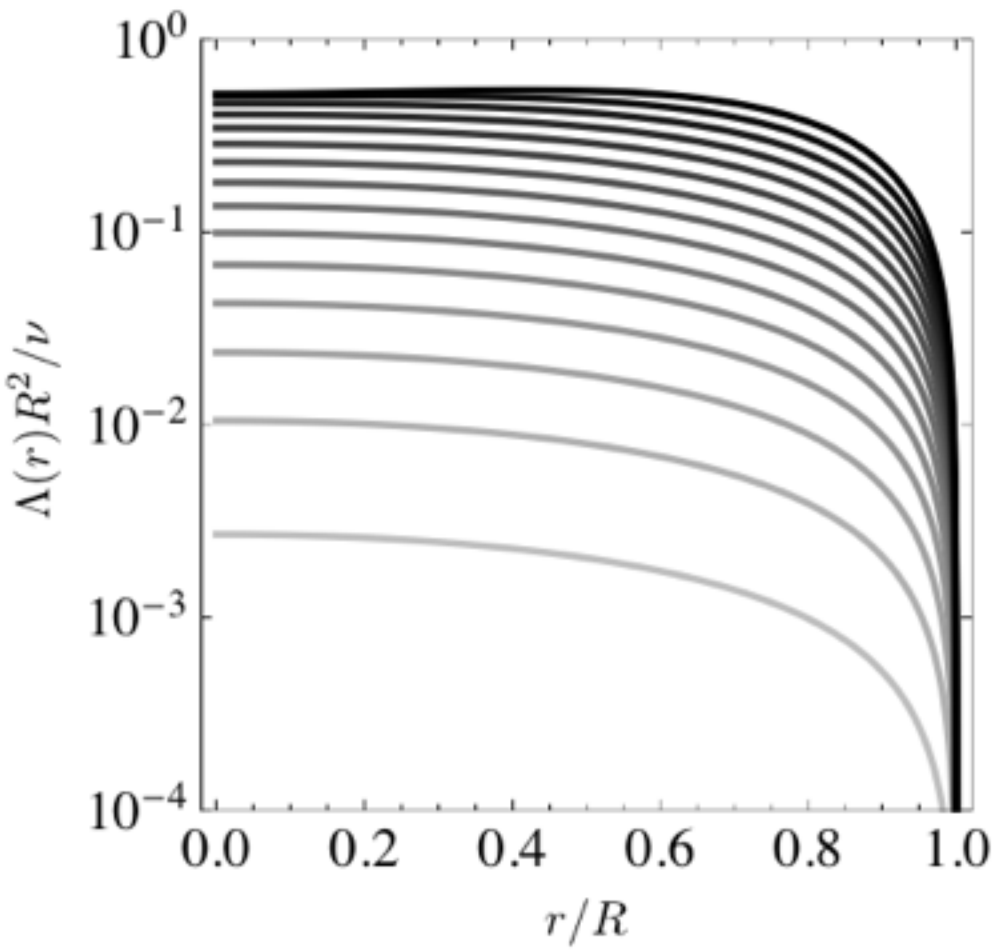
$$\mu = f(U^\alpha U^\beta h_{\alpha\beta}) = 1 + U^\alpha U^\beta h_{\alpha\beta} = 1 + U^\alpha U^\beta (g_{\alpha\beta} - \eta_{\alpha\beta}) = \frac{1}{A(r)}$$

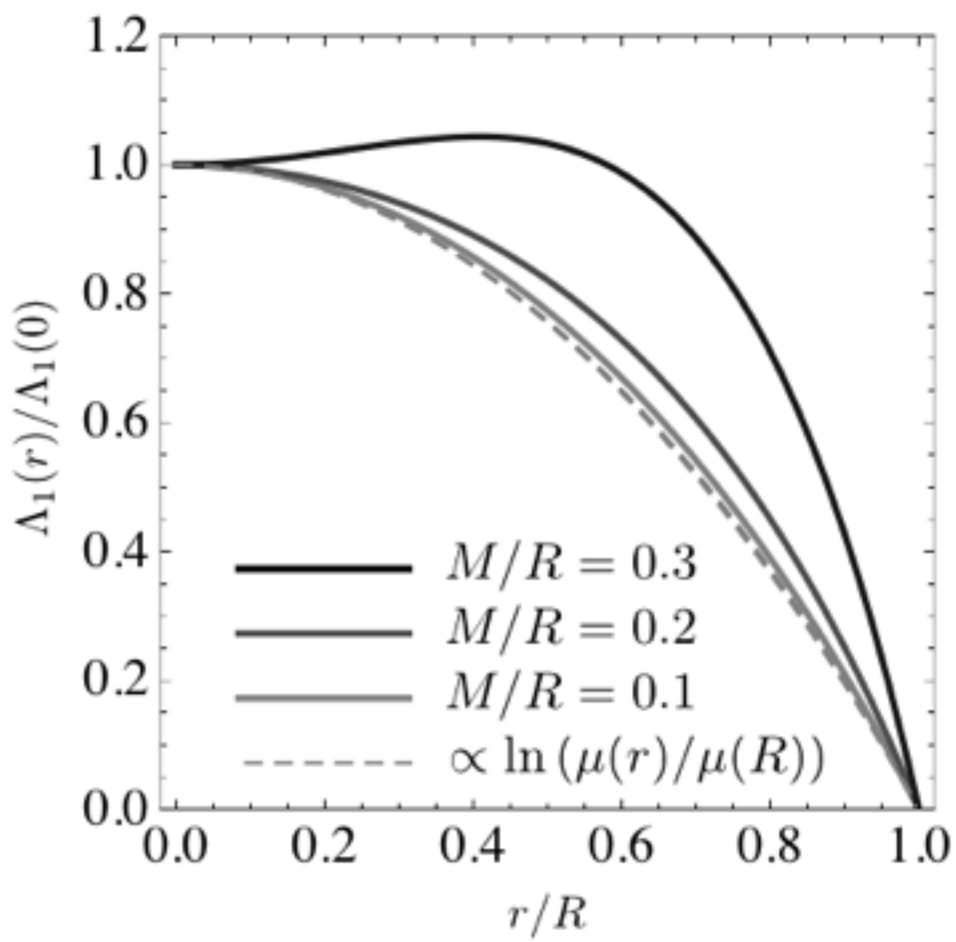
Perturbative solution about a constant density "star"

The solution for Lambda reads (with the boundary condition $\Lambda(R)=0$),

$$\Lambda(r) = \frac{24M\nu}{R^3}\left[\ln\frac{2-4M/R}{4+(r^2/R^2-9)M/R} + 2\,\mathrm{arctanh}\left(\frac{\sqrt{1-2Mr^2/R^3}}{3\sqrt{1-2M/R}}\right)\right] +$$

$$+\frac{36M\nu}{R^3}\,\frac{1+\left(r^2/R^2-3\right)M/R-\sqrt{(1-2M/R)\left(1-2Mr^2/R^3\right)}}{4+\left(r^2/R^2-9\right)M/R}+O(\nu^2).$$





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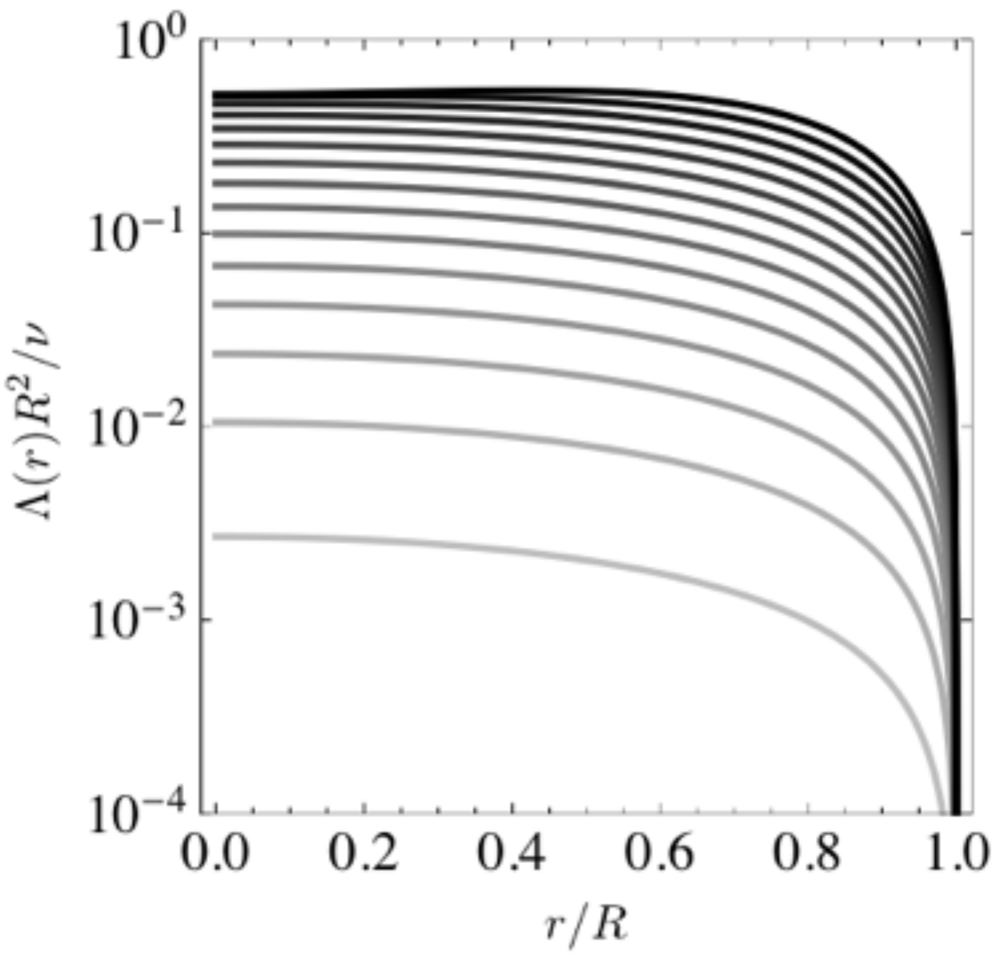
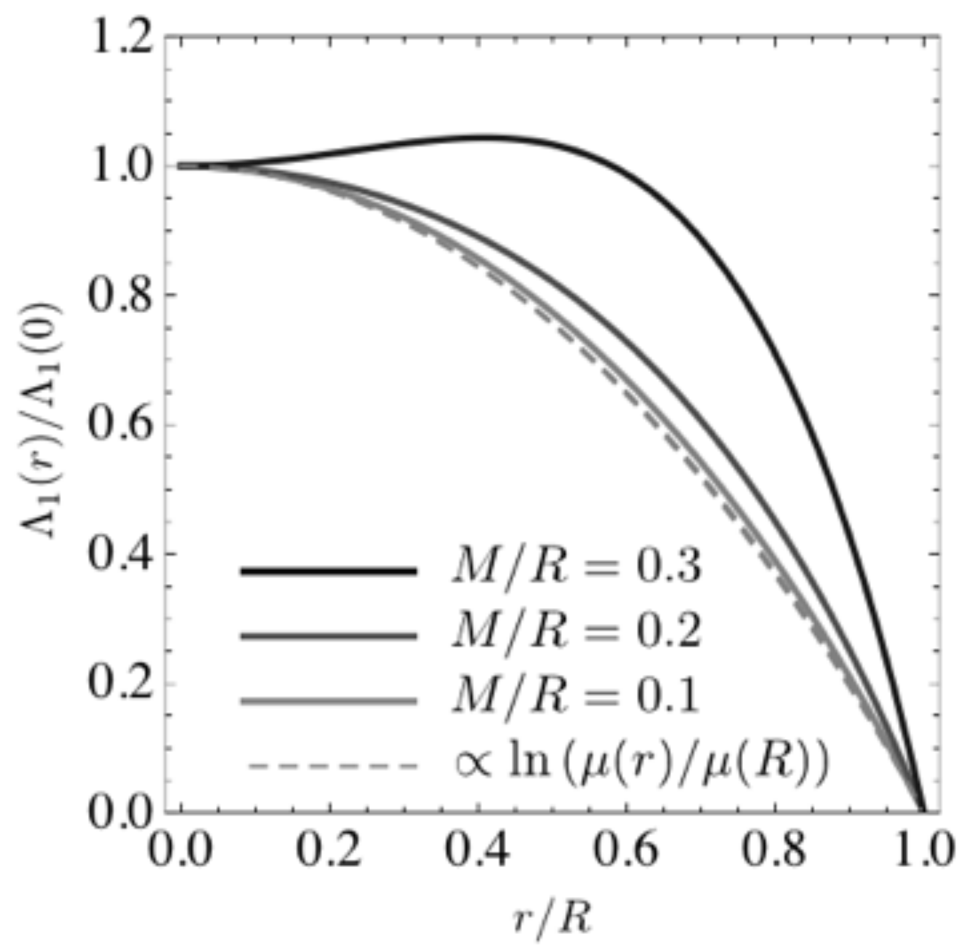




1. The induced Lambda is close to a constant inside the constant density medium.

2. The relation between Lambda and mu is not of the Log type.

Can a standard ST theory generate the same solutions?

Yes, it is possible to derive each one of them. But one should consider the following:

1. Introduce a perturbation approach that maps into the one used for the RG approach.

2. The procedure is the inverse one: in the standard ST picture one starts from the knowledge of the potential (which corresponds to Lambda) and derive the solution of the scalar field (which corresponds to either G or mu).

3. The potential in the ST picture would be an unusual one. It is indeed a function of the scalar field, but it depends on constants that describe the matter part (i.e., it is system dependent).

Conclusions

- The running of G and Λ at large scales, motivated by the RG, are currently considered from diverse approaches.

- We considered in particular the RG improved action framework in its most economic form (no kinetic terms for G) [Reuter, Weyer, PRD 2004].

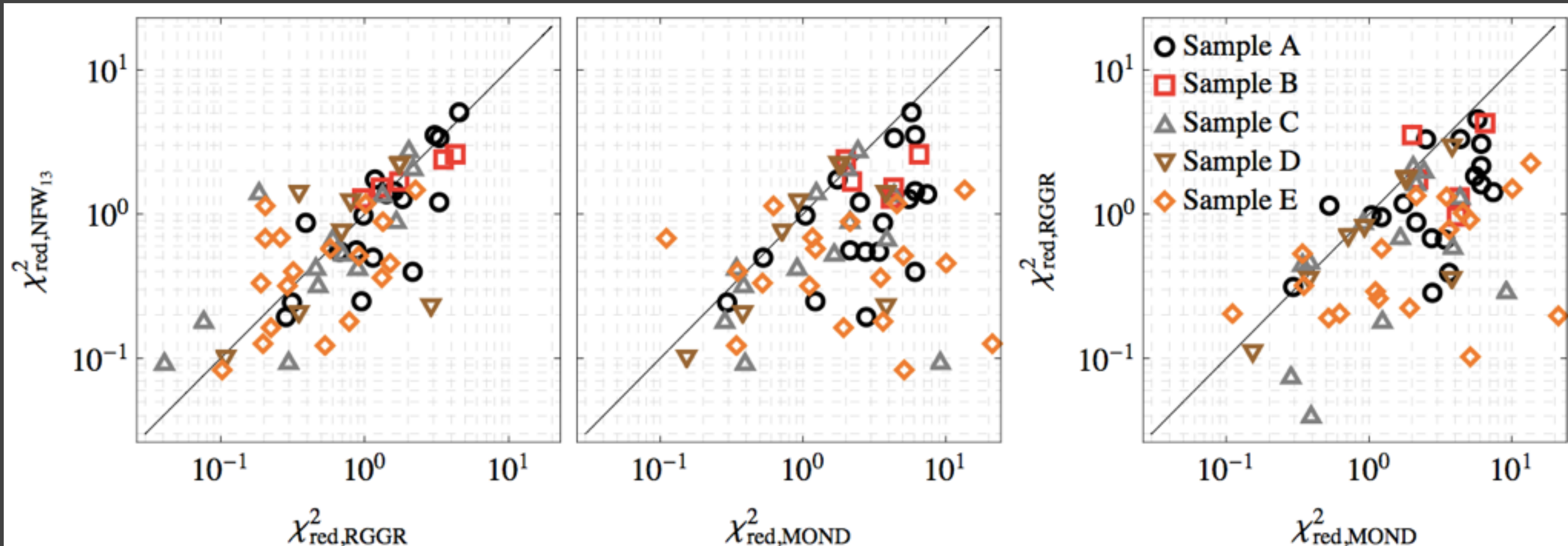
- If one considers that the RG flows of both G and Λ are independent on the matter fields, then the scale setting is derived and there is no need to use external fields and the theory becomes (locally equivalently to) f(R) gravity, as advocated by [Koch, Ramirez CQG 2011, and others].

- But one may consider that the flow of G and the scale setting are fixed from the start, while the Lambda is derived from the field equations. [DCR, Chauvineau, Piattella, 1504.05119]. In this case there is no equivalence with f(R).

- The RG improved action approach can be expressed as a ST theory, but a rather unusual one, both due to the potential, which is system dependent, and the perturbations. It can also be expressed as GR with nonminimal coupling to matter (with energy-momentum conservation).

Thank you!

A brief advertisement on another work…



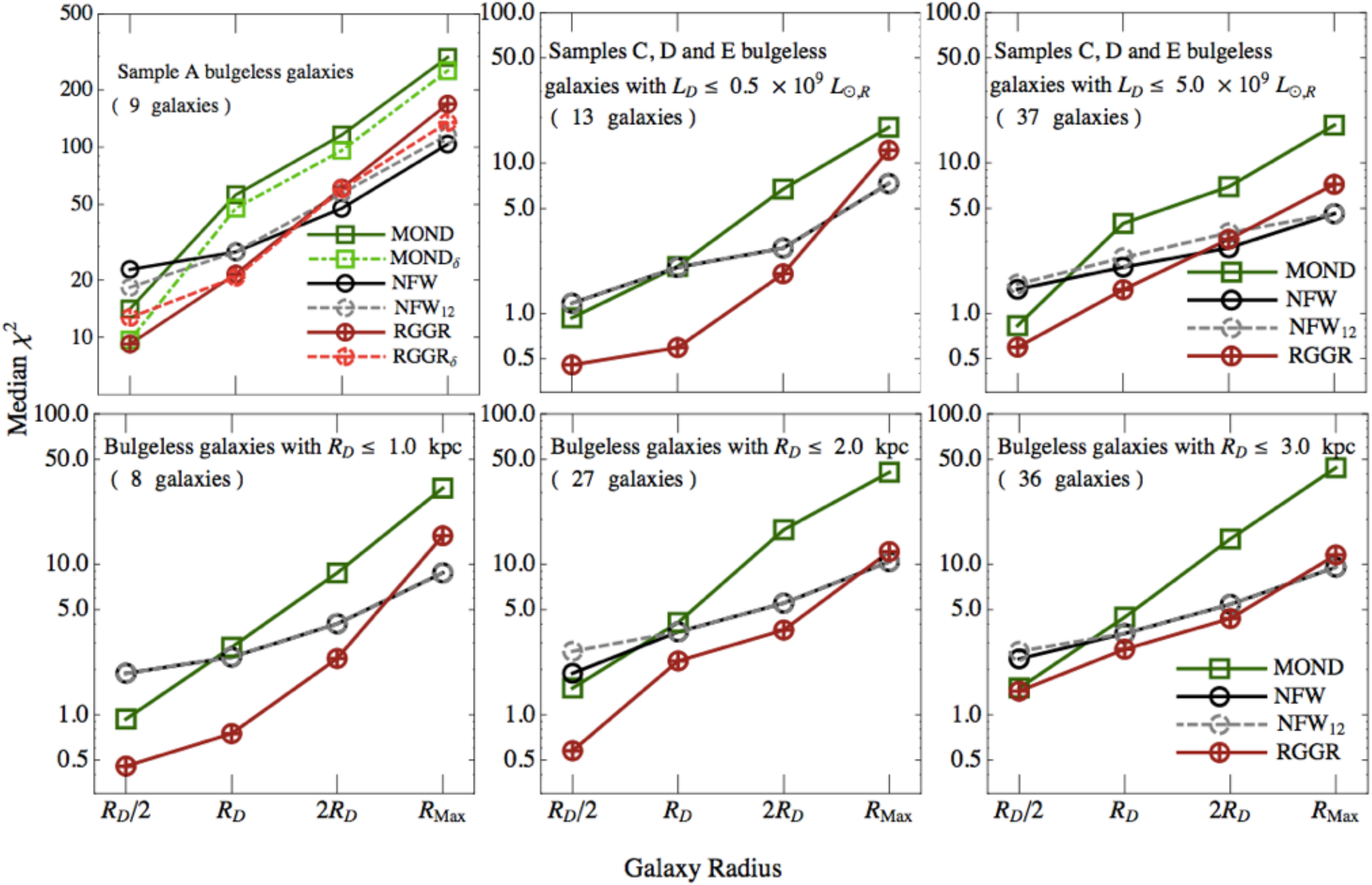
From [DCR, Oliveira, Fabris, Gentile, MNRAS 2014]

We studied 62 disk galaxies within 7 different models each, either in the context of dark matter or modified gravity.

And we developed a new test for comparing systematics as a function of the galaxy radius (in particular to test cusp/core-like effects)

Model	$\widetilde{\chi^2_{\rm red}}$	χ^2
MOND	2.1	8803
NFW	0.7	2967
RGGR	0.9	3707

The evolution of χ^2_R — subsamples without "larger" galaxies



Thank you again!