# Eternal Warm Inflation ? [1]

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# Cosmological Inflation[2]

Defined as an early accelerated expansion dominated by vacuum energy density and driven by a fluid with negative pressure:

$$\ddot{a} > 0, \quad p < -\rho/3 \quad \Rightarrow \quad \text{Inflation}$$

$$ds^2 = -dt^2 + e^{2Ht} dx^2, \qquad H \simeq \text{cte}$$

- Solves:
  - horizon problem
  - flatness problem
  - monopole problem (and other relic problems)
  - i.e., explains why the Universe is so large, so flat, and so empty
- Predicts:
  - scalar fluctuations in the CMB temperature
    - nearly but not exactly scale-invariant ($> 5\sigma$ !)
    - approximately Gaussian (at the $10^{-4}$ level !)

[2] (Starobinsky '80; Guth '81; Albrecht, Steinhardt '82; Linde '82)

# Different Pictures of Inflation: Cold and Warm Inflation

Cold Inflation:

Inflaton interactions with other d.o.f. only important during reheating and considered negligible during inflation:

$$\dot{\rho}_r + 4H\rho_r = 0, \quad (H = \dot{a}/a \sim cte)$$

$\Downarrow$

The radiation density during inflation redshifts away: $\rho_r \sim 1/a^4$

$\Downarrow$

Universe supercools during inflation

$\Rightarrow$ density perturbations sourced by inflaton's quantum fluctuations

$\Rightarrow$ Radiation must be produced (connection with the radiation dominated cosmological regime): decay of the inflaton's vacuum energy

Inflation $\Longrightarrow$ Reheating $\Longrightarrow$ Radiation $\Longrightarrow$ Matter

# Warm Inflation[3]:

$$\text{Inflaton} \Longrightarrow \text{Decay} \Longrightarrow \text{Radiation} \Longrightarrow \text{Matter}$$

Interactions of the inflaton with other d.o.f. are important during inflation, generate dissipation/viscosity terms $\Rightarrow$ small fraction of vacuum energy density can be converted to radiation

$$\dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\phi}^2$$

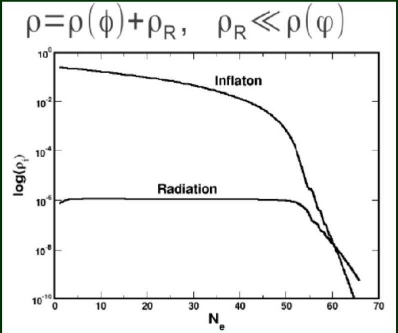
$\Rightarrow$ The radiation density during inflation might stabilize:

$$\Rightarrow \rho_r \sim \Upsilon\dot{\phi}^2/(4H)$$

$\Rightarrow$ Adiabatic curvature perturbations sourced by thermal fluctuations

[3] Ian G. Moss PLB154 1985, Yokoyama and Maeda PLB207 1988, A. Berera and L. Z. Fang PRL74 1995, Berera PRL75 1995

# Warm Inflation




$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V_{,\phi} = 0$$

Extra friction term due to interactions of the inflaton with other d.o.f.

$$\dot{\rho}_R + 4H\rho_R = \Upsilon\dot{\phi}^2, \quad \text{"Source term"}$$

Extra friction term: $Q = \frac{\Upsilon}{3H}$.

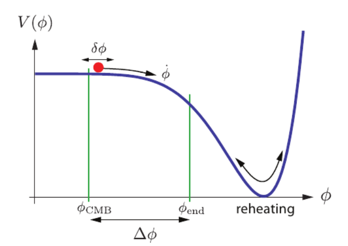
- $Q \ll 1, \quad T \ll H \implies$ Standard Cold Inflation
- $Q < 1, \quad T > H \implies$ Weak Dissipative Regime of Warm Inflation
- $Q > 1, \quad T > H \implies$ Strong Dissipative Regime of Warm Inflation

$\Rightarrow$ allow for smaller $\phi$ values during inflation:

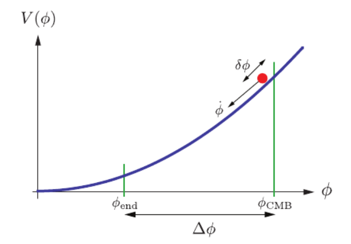
- smaller inflationary scale $\implies$ lower tensor-to-scalar ratio $(r)$

# Models of Inflation: Forms of potentials for the inflaton

Small field models:




Large field models:




- Small field models (Starobinsky modified gravity $R^2$ inflation model, Higgs inflation, string motivated inflation models, etc), they all predict a small tensor-to-scalar ratio $r \ll 0.1$.
- Large field models (power law $\phi^p$ ($p \geq 2$ potentials), chaotic inflation, etc), predict a large tensor-to-scalar ratio $r > 0.1$.

# *Planck* 2015 results. XX. Constraints on inflation

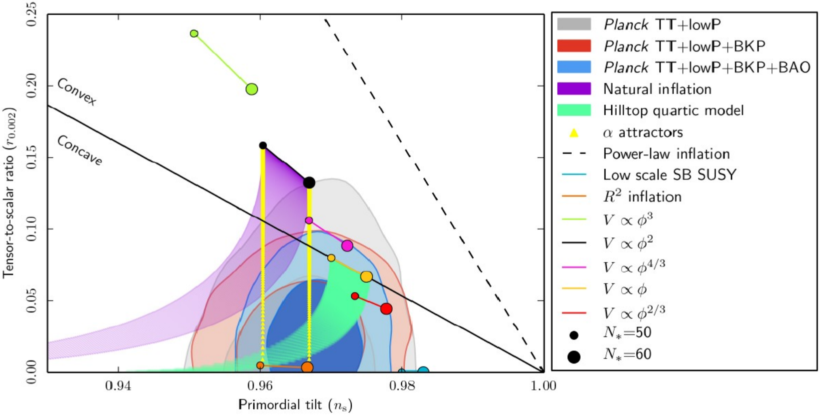



**Fig. 54.** Marginalized joint 68 % and 95 % CL regions for $n_s$ and $r_{0.002}$ from *Planck* alone and in combination with its cross-correlation with BICEP2/Keck Array and/or BAO data compared with the theoretical predictions of selected inflationary models.

$r_{0.002} < 0.10$ (95 % CL, *Planck* TT+lowP+BKP)    $n_s = 0.9667 \pm 0.0066$ (68 % CL)

# *Planck* 2015 results. XX. Constraints on inflation

## Power-law potentials

$$V(\phi) = \lambda M_{\rm pl}^4 \left(\frac{\phi}{M_{\rm pl}}\right)^n ,$$

**Table 6.** Results of the inflationary model comparison. We provide $\Delta\chi^2$ with respect to base ΛCDM and Bayes factors with respect to $R^2$ inflation.

| Inflationary model | $\Delta\chi^2$ | | $\ln B_{0X}$ | |
|---|---|---|---|---|
| | $w_{\rm int} = 0$ | $w_{\rm int} \neq 0$ | $w_{\rm int} = 0$ | $w_{\rm int} \neq 0$ |
| $R + R^2/(6M^2)$ | +0.8 | +0.3 | ... | +0.7 |
| $n = 2/3$ | +6.5 | +3.5 | −2.4 | −2.3 |
| $n = 1$ | +6.2 | +5.5 | −2.1 | −1.9 |
| $n = 4/3$ | +6.4 | +5.5 | −2.6 | −2.4 |
| $n = 2$ | +8.6 | +8.1 | −4.7 | −4.6 |
| $n = 3$ | +22.8 | +21.7 | −11.6 | −11.4 |
| $n = 4$ | +43.3 | +41.7 | −23.3 | −22.7 |
| Natural | +7.2 | +6.5 | −2.4 | −2.3 |
| Hilltop ($p = 2$) | +4.4 | +3.9 | −2.6 | −2.4 |
| Hilltop ($p = 4$) | +3.7 | +3.3 | −2.8 | −2.6 |
| Double well | +5.5 | +5.3 | −3.1 | −2.3 |
| Brane inflation ($p = 2$) | +3.0 | +2.3 | −0.7 | −0.9 |
| Brane inflation ($p = 4$) | +2.8 | +2.3 | −0.4 | −0.6 |
| Exponential inflation | +0.8 | +0.3 | −0.7 | −0.9 |
| SB SUSY | +0.7 | +0.4 | −2.2 | −1.7 |
| Supersymmetric $\alpha$-model | +0.7 | +0.1 | −1.8 | −2.0 |
| Superconformal ($m = 1$) | +0.9 | +0.8 | −2.3 | −2.2 |
| Superconformal ($m \neq 1$) | +0.7 | +0.5 | −2.4 | −2.6 |

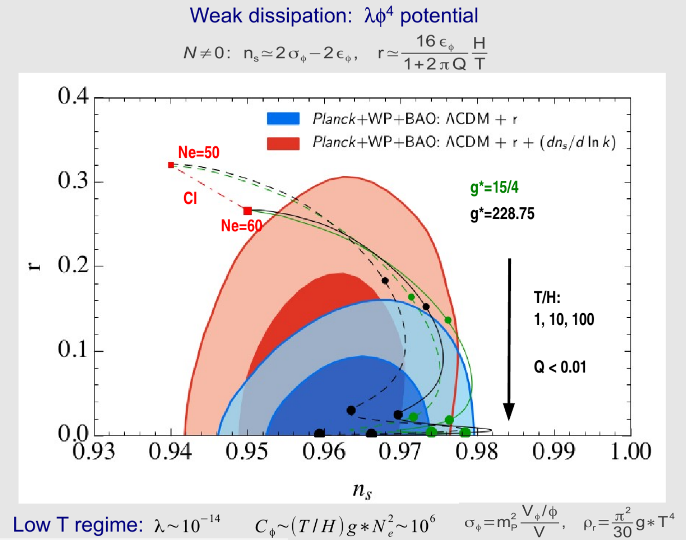
**Table 17.** Results of inflationary model comparison using the cross-correlation between BICEP2/Keck Array and *Planck*. This table is the analogue to Table 6, which did not use the BKP likelihood.

| Inflationary Model | $\ln B_{0X}$ | |
|---|---|---|
| | $w_{\rm int} = 0$ | $w_{\rm int} \neq 0$ |
| $R + R^2/6M^2$ | ... | +0.3 |
| $n = 2$ | −6.0 | −5.6 |
| Natural | −5.6 | −5.0 |
| Hilltop ($p = 2$) | −0.7 | −0.4 |
| Hilltop ($p = 4$) | −0.6 | −0.9 |
| Double well | −4.3 | −4.2 |
| Brane inflation ($p = 2$) | +0.2 | 0.0 |
| Brane inflation ($p = 4$) | +0.1 | −0.1 |
| Exponential inflation | −0.1 | 0.0 |
| SB SUSY | −1.8 | −1.5 |
| Supersymmetric $\alpha$-model | −1.1 | +0.1 |
| Superconformal ($m = 1$) | −1.9 | −1.4 |

## Hilltop models

$$V(\phi) \approx \Lambda^4 \left(1 - \frac{\phi^p}{\mu^p} + ...\right)$$

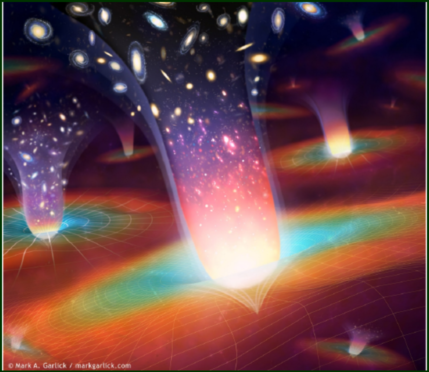
# Trajectories in the $(n_s, r)$ plane for $\lambda\phi^4$ potential[4]



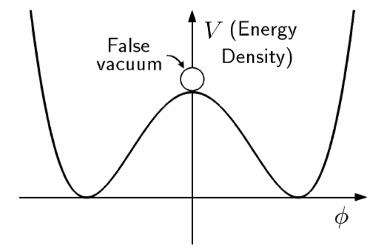

[4] S. Bartrum, M. Bastero-Gil, A. Berera, R. Cerezo, ROR and J. G. Rosa, PLB 732 (2014) 116

# Eternal Inflation and the Multiverse

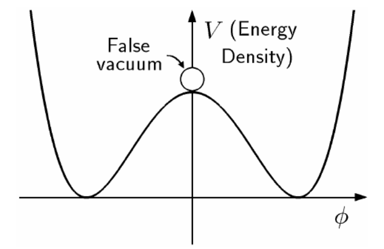
- Current fundamental theories do not predict a unique vacuum.
- False vacuum decay can lead to inflation.
- False vacuum decay can happen in different parts of the universe.
- Pockets of inflation universe are constantly been created.
- Strongly motivates that we inhabit an eternally inflating universe.



False vacuum decay can happen in different parts of the universe:



False vacuum decay can happen in different parts of the universe:



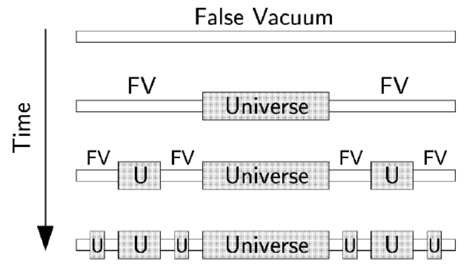
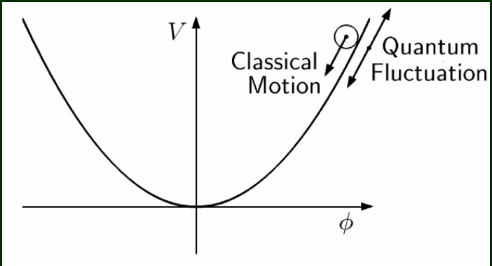
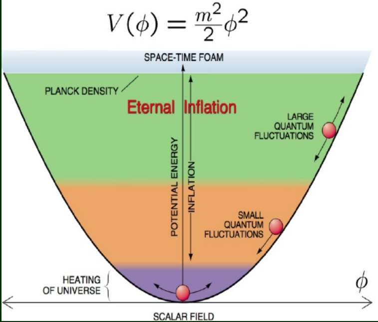




Figure 3: A schematic illustration of eternal inflation.

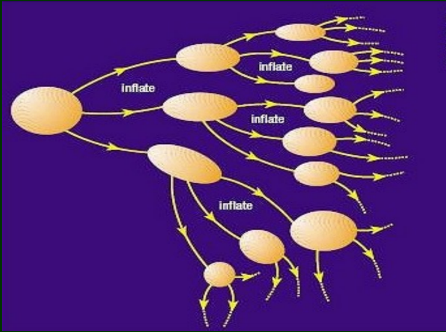



Occasional quantum jumps in localized regions can raise the scalar field to a higher value of the energy density, leading to regions of faster inflation.

Consider any particular time-like geodesic line at the stage of inflation. One can show that for any given observer following this geodesic, the duration $t_i$ of the stage of inflation on this geodesic will be finite.

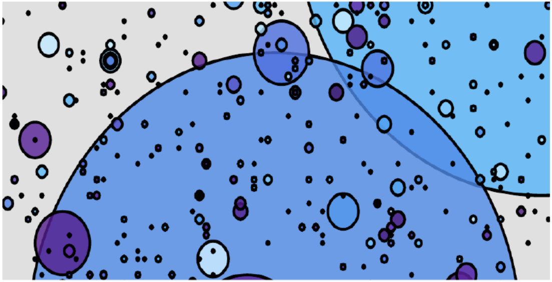
One the other hand, eternal inflation implies that if one takes all such geodesics and calculate the time $t_i$ for each of them, then there will be no upper bound for $t_i$ , i.e. for each time $T$ there will be such geodesic which experience inflation for the time $t_i > T$.

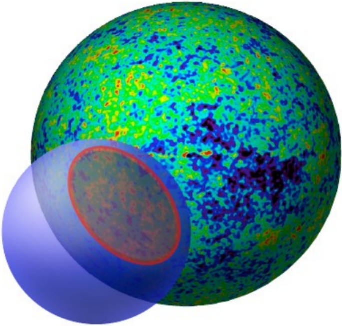


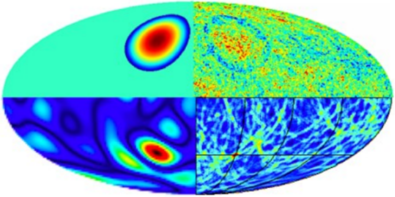

8: Eternal inflation predicts never-ending reproduction of inflating regions.

# Observational tests:

The collision of our bubble with others provides an observational test of eternal inflation.

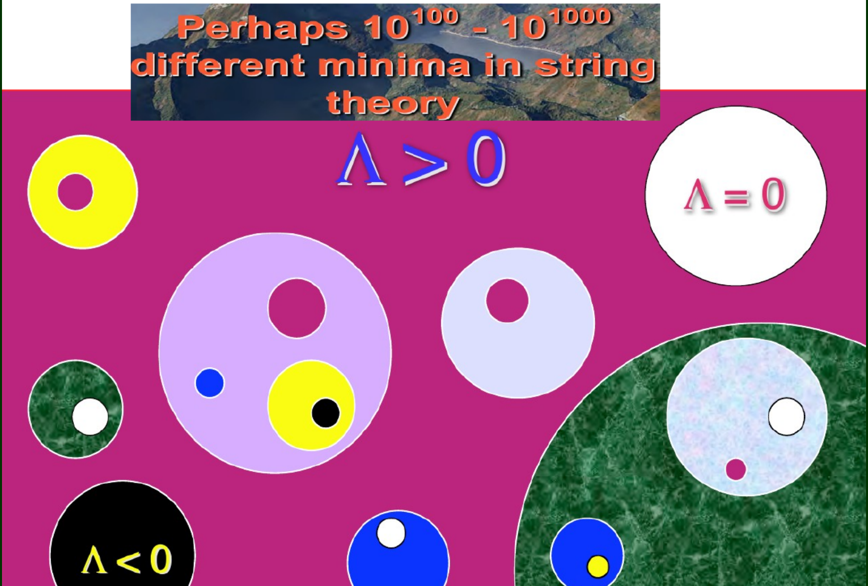






Zhang and Johnson, arXiv:1501.00511;
Wainwright et. al., JCAP, 030 (2014);
Feeney, et. al., PRL 107, 071301 (2011);
Aguirre and Johnson, Rep. Progr. Phys. 74, 074901 (2011).

# String Theory Multiverse



# Is eternal inflation (or the multiverse) a problem ?


The Inflation Debate
Is the theory at the heart of modern cosmology deeply flawed?
*By Paul J. Steinhardt*
2011 Scientific American

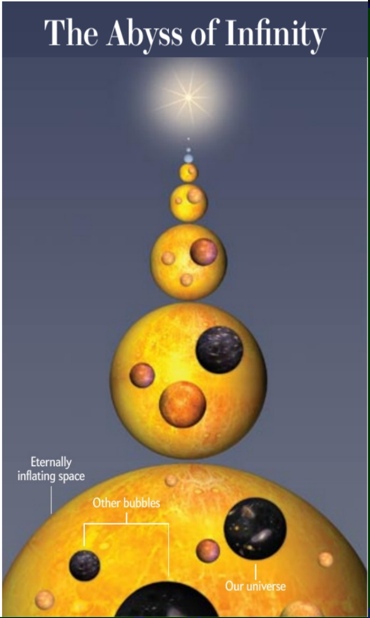
" … an infinite number of bubbles form with an infinite variety of properties. Everything that can happen does happen in some bubble. A theory that predicts everything predicts nothing."

"Inflationary paradigm in trouble after Planck2013"!
Ijjas, Steinhardt, Loeb 1304.2785
**The curse of the multiverse:** "After Planck, eternal inflation becomes unavoidable, which is a disaster.!"

Strong Responses:

Inflationary paradigm after Planck 2013
Guth, Kaiser, Nomura, arXiv:1312.7619

Inflationary Cosmology after Planck 2013
Andrei Linde, arXiv:1402.0526



# Is there an eternal warm inflation ?

1) Start from effective stochastic equation for the inflaton in WI:

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} - \frac{1}{a^2}\nabla^2\phi + V'(\phi) = \xi_q + \xi_T$$

$\xi_q$ and $\xi_T$ are stochastic Gaussian noise terms. In particular

$$\langle \xi_T(\mathbf{x}, t)\xi_T(\mathbf{x}', t')\rangle = 2\Upsilon T a^{-3}\delta(\mathbf{x} - \mathbf{x}')\delta(t - t')$$

2) Write the equivalent Fokker-Planck equation for the homogeneous inflaton field:

$$\frac{\partial}{\partial t}P(\varphi, t) = -\frac{\partial}{\partial \varphi}\left[D^{(1)}P(\varphi, t)\right] + \frac{\partial^2}{\partial \varphi^2}\left[D^{(2)}P(\varphi, t)\right]$$
$$= -\frac{\partial}{\partial \varphi}\left\{D^{(1)}P(\varphi, t) - \frac{\partial}{\partial \varphi}\left[D^{(2)}P(\varphi, t)\right]\right\}$$

$D^{(1)}$ and $D^{(2)}$ are the drift and diffusion coefficients:

$$D^{(1)} = -\frac{V_{,\varphi}(\varphi)}{3H(1+Q)}, \qquad Q = \frac{\Upsilon}{3H}$$

$$D^{(2)} = \frac{H^3\sqrt{1+4\pi Q/3}}{8\pi^2(1+Q)^2}\left[1 + 2n_k + \left(\frac{T}{H}\right)\frac{2\pi Q}{\sqrt{1+4\pi Q/3}}\right]$$

If diffusion dominates over drift, a fluctuation dominate regime (FDR) is present $\Rightarrow$ self-reproduction of H-regions occurs.

3) Transform the FP equation into a Sturm-Liouville problem:

$$\sigma \to \int \frac{d\varphi}{\sqrt{D^{(2)}(\varphi)}}$$

$$\frac{\partial}{\partial\varphi} \to \frac{1}{\sqrt{D^{(2)}(\varphi)}}\frac{\partial}{\partial\sigma}$$

$$P_n(\varphi) \to \frac{1}{D^{(2)}(\varphi)^{3/4}} \exp\left[\frac{1}{2}\int d\varphi \frac{D^{(1)}(\varphi)}{D^{(2)}(\varphi)}\right]\psi_n(\varphi)$$

$$\frac{\partial^2}{\partial\sigma^2}\psi_n(\sigma) - V_S(\sigma)\psi_n(\sigma) = \Lambda_n\psi_n(\sigma)$$

where the effective potential $V_S$ is defined in terms of the drift and diffusion coefficients:

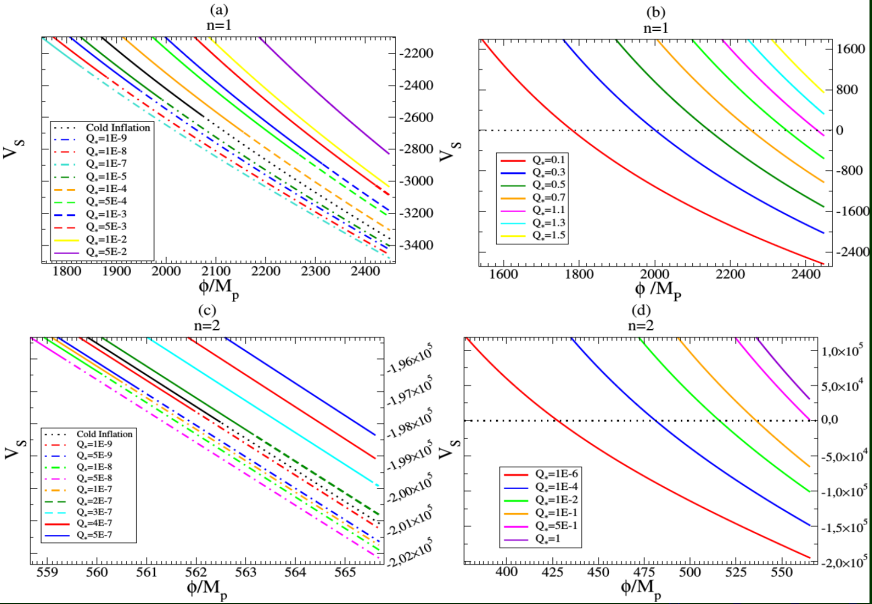
$$V_S(\sigma) = \frac{3}{16}\frac{(D^{(2)}_{,\varphi})^2}{D^{(2)}} - \frac{D^{(2)}_{,\varphi\varphi}}{4} - \frac{D^{(2)}_{,\varphi}D^{(1)}}{2D^{(2)}} + \frac{D^{(1)}_{,\varphi}}{2} + \frac{(D^{(1)})^2}{4D^{(2)}}$$

4) We can write for the volume weighted distribution:
$P_V(\phi, t) = \sum_n P_{V_n}(x)e^{\Lambda'_n t}$.
If $\Lambda'_0 > 0$ ($E'_0 < 0$), the physical volume of the inflating regions grows with time and eternal self-reproduction is present.

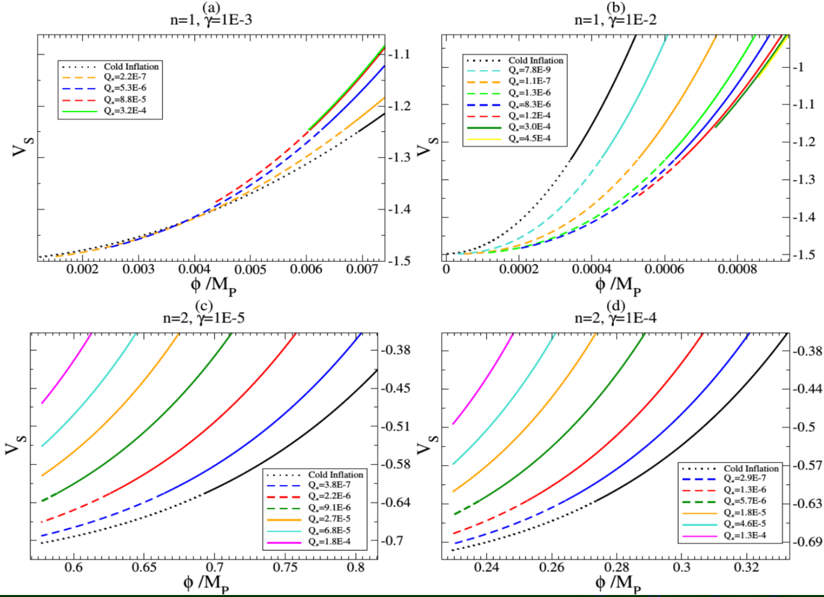
$$\Lambda_0 = -\min_{\psi(\sigma)}\frac{\int d\sigma\left[\left(\frac{d\psi_n}{d\sigma}\right)^2 + V_S(\sigma)\psi_n^2\right]}{\int d\sigma\ \psi_n^2}$$

5) $\Lambda_0$ is obtained by solving (numerically) the equation in WI.

$$V(\varphi) = V_0 \left( \frac{\varphi}{M_p} \right)^{2n} \quad , \quad V_0 = \lambda M_p^4/(2n)$$



$$V(\varphi) = V_0 \left[ 1 - \frac{|\gamma|}{2n} \left( \frac{\varphi}{M_p} \right)^{2n} \right] \quad , \quad V_0 = \lambda M_p^4/(2n)$$



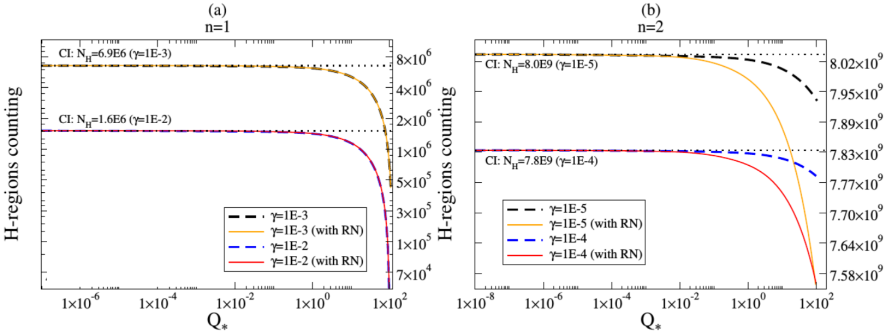



Figure 12: (Color online) The number of H-regions as a function of $Q_\star$, given a $N_e > N_{th}$ such that a self-reproduction regime is present in the hilltop case.

# Summary

- Inflation is the most strong and well supported scenario of early Universe solving the outstanding standard Big-Bang cosmological model problems
- Two pictures: Cold and Warm Inflation. Warm inflation fully includes the effects of interactions in the inflaton dynamics. Dynamics that emerges is naturally dissipative (as most of the dynamics of systems in Nature !).
- Dissipative effects due to decaying fields during inflation can strongly influence the dynamics and modify the inflationary predictions ($r$, $n_s$, $f_{NL}$, etc).
- Inflation is predicted to be eternal. Once inflation starts, it can self-reproduce indefinitely.

# Summary

- Warm inflation can also be eternal, but dissipative effects can suppress the self-reproduction of inflating H-regions.
- Eternal inflation might even be suppressed entirely if $n_s' < 0$ (in warm inflation[5], $n_s' = -(n_s - 1)^2 \simeq -0.0016$).
- In Warm inflation: larger number of efolds and larger (chaotic) or much smaller field amplitudes (hilltop) required for initiating EI.

[5] S. Bartrum, M. Bastero-Gil, A. Berera, R. Cerezo, ROR and J. G. Rosa, PLB 732 (2014) 116