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# Probing the slow-roll approximation in single-field inflation with the Planck data

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# Outline

- Review of the single-field slow-roll Inflation idea
- Examples of models that violate the slow-roll approximation
- The Generalized Slow-Roll technique
- Application I: steps in single-field inflation
- Application II: model independent search for slow-roll violation in single-field inflation

# Bibliography


1. arXiv:1411.5956 [pdf, other]

   **Polarization Predictions for Inflationary CMB Power Spectrum Features**
   Vinícius Miranda (KICP), Wayne Hu (KICP), Cora Dvorkin (Harvard University)
   Comments: 10 pages, 8 figures, submitted to PRD
   Journal-ref: Phys. Rev. D 91, 063514 (2015)
   Subjects: **Cosmology and Nongalactic Astrophysics (astro-ph.CO)**

2. arXiv:1403.5231 [pdf, other]

   **Steps to Reconcile Inflationary Tensor and Scalar Spectra**
   Vinícius Miranda, Wayne Hu, Peter Adshead
   Comments: Revisions match published version with more extended analysis that leave qualitative conclusions unchanged
   Journal-ref: Phys. Rev. D 89, 101302 (2014)
   Subjects: **Cosmology and Nongalactic Astrophysics (astro-ph.CO)**

3. arXiv:1312.0946 [pdf, other]

   **Inflationary Steps in the Planck Data**
   Vinicius Miranda, Wayne Hu
   Comments: submitted
   Journal-ref: Phys. Rev. D 89, 083529 (2014)
   Subjects: **Cosmology and Nongalactic Astrophysics (astro-ph.CO)**

4. arXiv:1303.7004 [pdf, other]

   **Bispectrum in Single-Field Inflation Beyond Slow-Roll**
   Peter Adshead, Wayne Hu, Vinícius Miranda
   Comments: 20 pages, 6 figures
   Journal-ref: Phys. Rev. D88 023507 (2013)
   Subjects: **Cosmology and Nongalactic Astrophysics (astro-ph.CO)**

5. arXiv:1207.2186 [pdf, other]

   **Warp Features in DBI Inflation**
   Vinicius Miranda, Wayne Hu, Peter Adshead (U. Chicago)
   Comments: 10 pages, 5 figures, submitted to PRD
   Journal-ref: Phys. Rev. D86 063529 (2012)
   Subjects: **Cosmology and Nongalactic Astrophysics (astro-ph.CO)**; High Energy Physics - Theory (hep-th)


# Single-Field Slow-Roll Inflation

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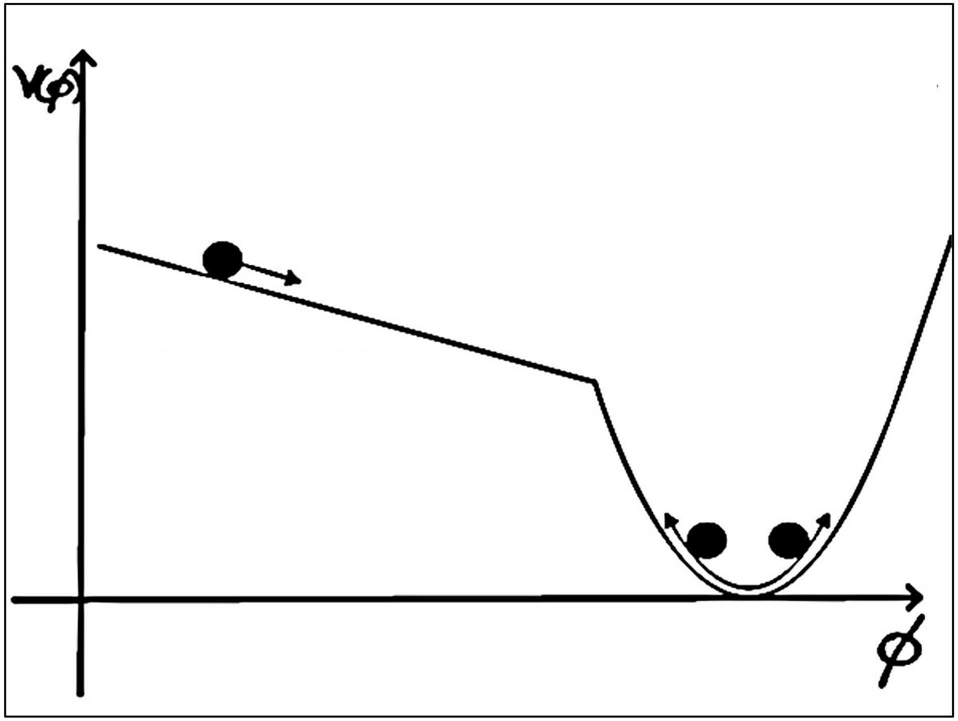
# Single-Field Slow-Roll Inflation
## Background Evolution

Single clock: field position




- Friction-dominated motion

$$\frac{d\phi}{dt} \approx -\sqrt{\frac{V(\phi)}{3}}\frac{V_{\phi}(\phi)}{V(\phi)}$$

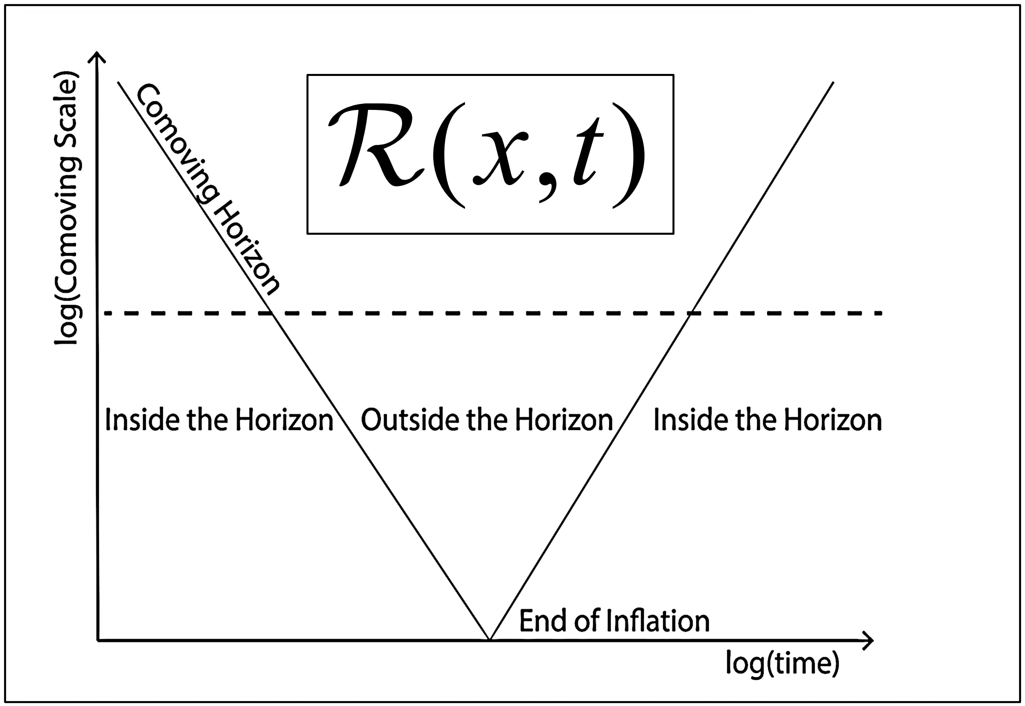
- Quasi-de Sitter expansion

$$\left|\frac{d\ln H}{d\ln a}\right| \ll 1$$

# Single-Field Slow-Roll Inflation

## Curvature Perturbations

conserved outside the horizon




- Scale-Factor

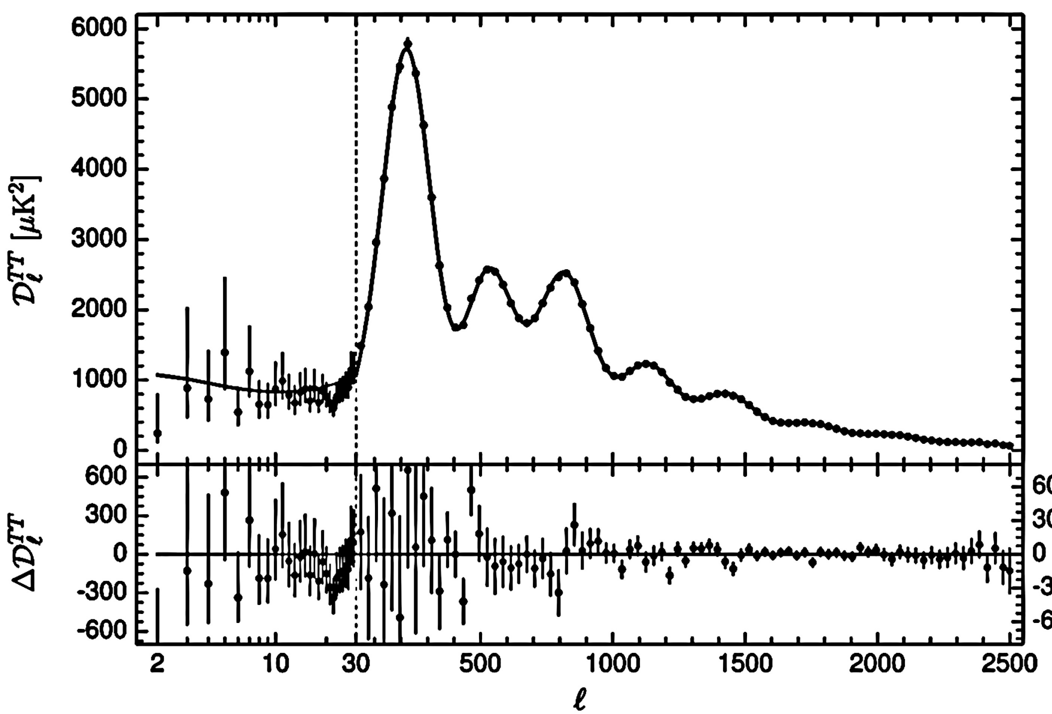
$$\boxed{a^2(x,t) = a^2(t)e^{2\mathcal{R}(x,t)}}$$

- Primordial power-spectrum

$$\boxed{\Delta^2_{\mathcal{R}_k} = \left|\mathcal{R}_k\right|^2}$$

<u>Seeds of the perturbations seen in the CMB</u>

# Single-Field Slow-Roll Inflation
## Primordial power-spectrum: power Law



Ade et al, Planck 2015

$$\Delta^2_{\mathcal{R}_k} = A_s\left(\frac{k}{k_p}\right)^{n_s-1}$$

$$k_p = 0.05\,\mathrm{Mpc}^{-1}$$

$$1 - n_s \sim 0.03$$

$$A_s \sim 2.2\times 10^{-9}$$

Amplitude constrained around l ~ 1000. Tilt also constrained w/ high-l data

# Single-Field Slow-Roll Inflation
## Primordial power-spectrum: power Law

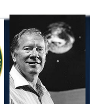
Planck data is very sensitive to broad-band changes in the primordial power-spectrum.

$$\boxed{\ln(10^{10} A_s) = 3.094 \pm 0.034}$$

$$\boxed{n_s = 0.9645 \pm 0.0049}$$

TT+TE+EE+LOW-P, Planck 2015

<u>Are there interesting single-field models whose power-spectrum cannot be described by a power law?</u>



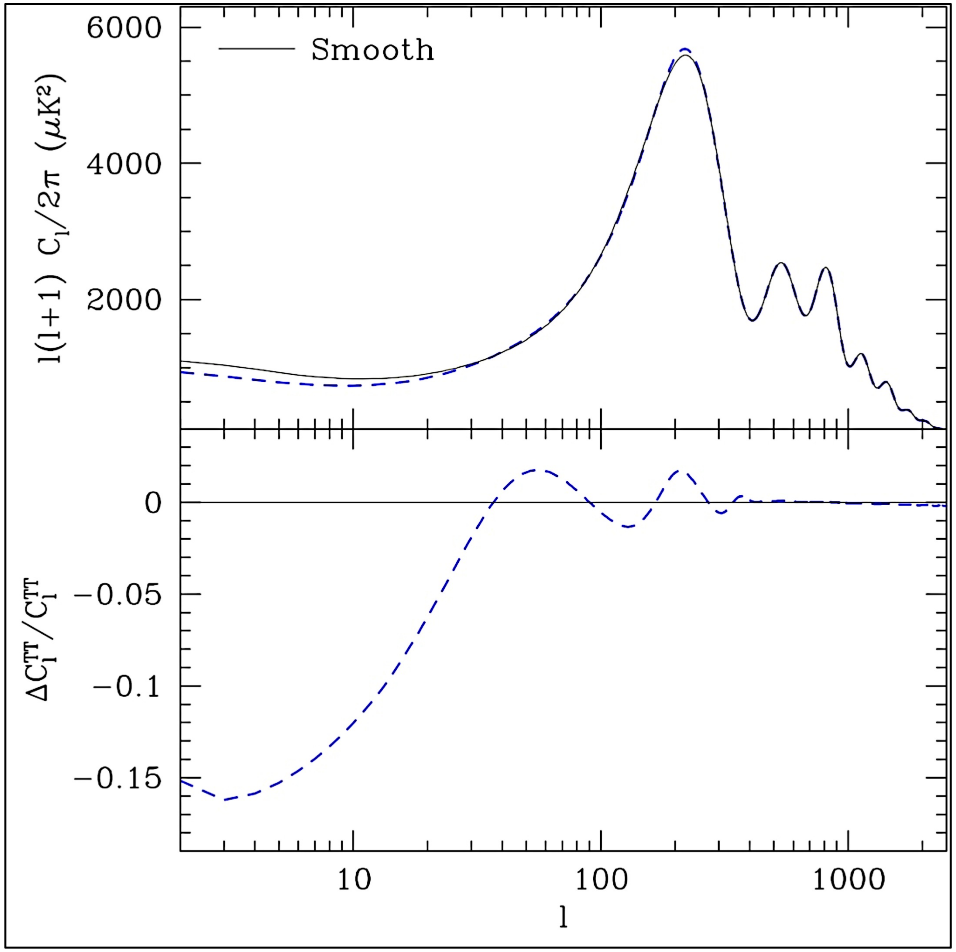
# Single Field Slow-Roll Inflation
## violation of the slow-roll approximation

In fact there are interesting models w/ rich phenomenology. Hints from WMAP and Planck 2013

- Steps in the Inflaton potential — Dvorkin & Hu (2009), Adshead et al (2011, 2012, 2013), Miranda et al (2011), Miranda & Hu (2012, 2014)
- Monodromy Inflation — Flauger et al (2009, 2014), Peiris et al (2013), Meerburg (2014), Meerburg et al (2014), Motohashi & Hu (2015)
- Large running — Meerburg (2014)
- Cut off Inflation — Contaldi et al. (2003)

# Breaking Slow-Roll

## Steps: transient violation of slow-roll



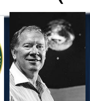

Miranda et al (2014)

New phenomenology

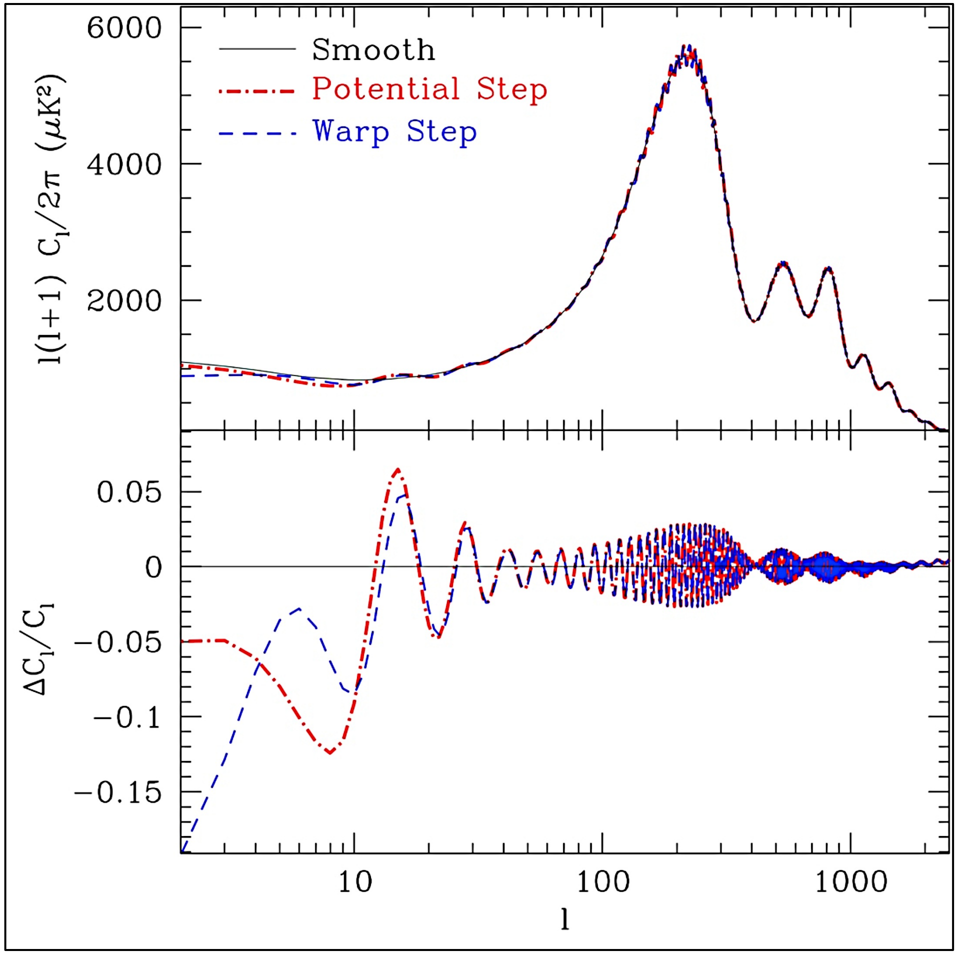
$$\Delta^2_{\mathcal{R}_k} = A_s \left(\frac{k}{k_*}\right)^{n_s-1}$$

End inflation?

No, if <u>transient</u>!



# Breaking Slow-Roll
## Steps: transient violation of slow-roll




Miranda & Hu (2014)

New phenomenology

$$\Delta^2_{\mathcal{R}_k} = A_s\left(\frac{k}{k_*}\right)^{n_s-1}$$

End inflation?

No, if <u>transient</u>!

# Breaking Slow-Roll

## Steps: transient violation of slow-roll

- Temperature-Spectrum
- Polarization
- Cross-Spectrum
- Primordial Non-Gaussianity
- Lensing Reconstruction

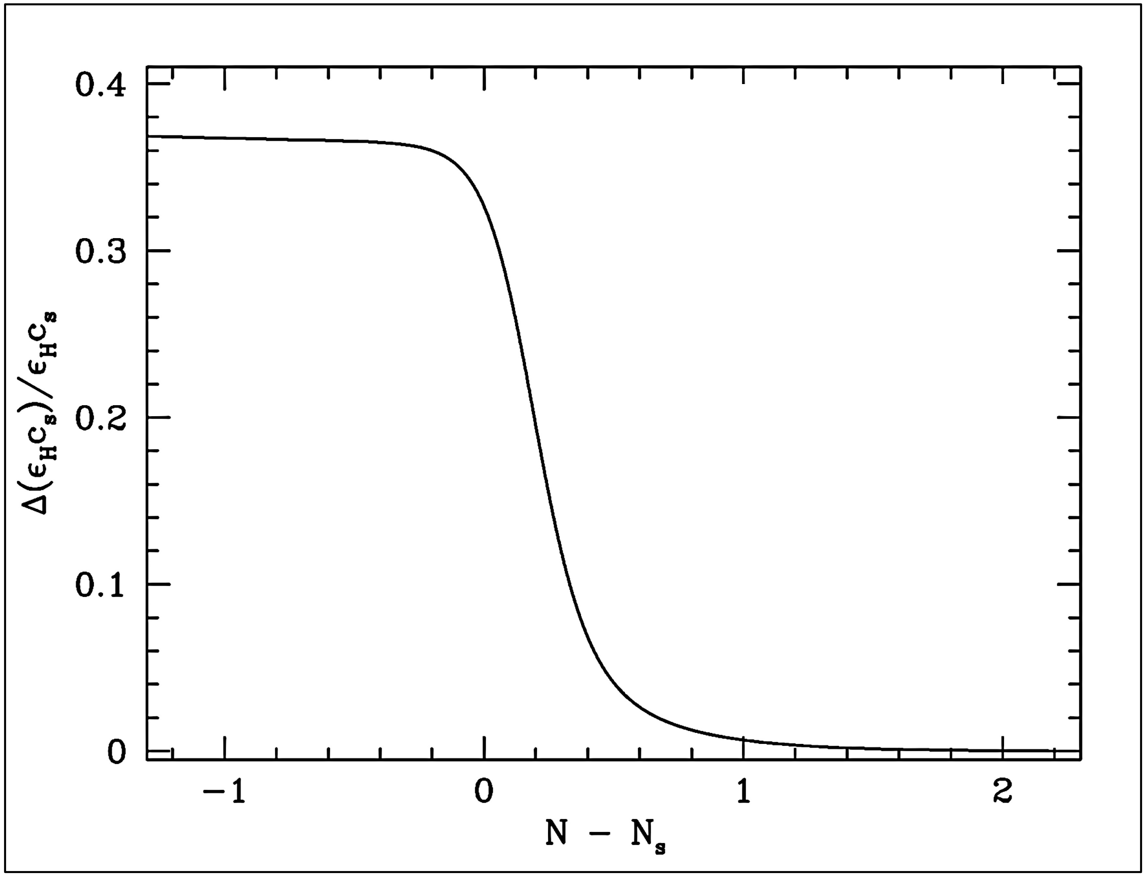
FIT

Prediction

Adshead et al (2011), Adshead & Hu (2012), Adshead et al (2012), Miranda & Hu (2014)

# Breaking Slow-Roll

## Steps: transient violation of slow-roll




Miranda et al (2014)

How the primordial power-spectrum can be computed in single-field models that violate slow-roll?

# Generalized Slow-Roll

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# Power-Spectrum

## Mukhanov-Sasaki equation

$$\boxed{\frac{d^2\mathcal{R}_k}{ds^2} - \frac{2}{s}\left(1 - \frac{f'}{f}\right)\frac{d\mathcal{R}_k}{ds} + k^2\mathcal{R}_k = 0}$$

<u>Sound horizon of perturbations</u>

$$\boxed{s = \int_{\ln a}^{0} \frac{c_s\, d\ln\tilde{a}}{\tilde{a}H}}$$

<u>Source</u>

$$\boxed{f^2 = 8\pi^2 \frac{\epsilon_H c_s}{H^2}\left(\frac{aHs}{c_s}\right)^2}$$

# Power-Spectrum

## Mukhanov-Sasaki equation

$$\boxed{\frac{d^2\mathcal{R}_k}{ds^2} - \frac{2}{s}\left(1 - \frac{f'}{f}\right)\frac{d\mathcal{R}_k}{ds} + k^2\mathcal{R}_k = 0}$$

Provides the evolution of curvature perturbations with respect to the sound horizon of perturbations

$$\boxed{s = \int_{\ln a}^{0} \frac{c_s\, d\ln\tilde{a}}{\tilde{a}H}}$$

# Power-Spectrum
## Mukhanov-Sasaki equation

- Can only handle case-by-case basis.
- Too slow for MCMC methods.

# Power-Spectrum
## Generalized Slow-Roll

- Valid for single-clock inflation.
- Enough precision for analysis w/ the Planck data.
- Framework for constructing analytical solutions.
- Framework for model independent constrains.

Stewart (2001), Dvorkin & Hu (2009), Hu (2011), Adshead et al (2012), Hu (2014)

# Power-Spectrum

## Generalized Slow-Roll

$$\boxed{\ln \Delta_{\mathcal{R}}^2 = I_0(k) + \ln[1 + I_1^2(k)]}$$

$$\boxed{I_j(k) \propto \int d \ln s \, W_j(ks) G'(\ln s)}$$

All phenomenology encoded in a single source

Dvorkin & Hu (2009), Hu (2011), Adshead et al (2012), Hu (2014), Motohashi & Hu (2015)

# Power-Spectrum

## Connection with Mukhanov-Sasaki source

$$\boxed{\frac{d^2\mathcal{R}_k}{ds^2} - \frac{2}{s}\left(1 - \frac{f'}{f}\right)\frac{d\mathcal{R}_k}{ds} + k^2\mathcal{R}_k = 0}$$

$$\boxed{G'(\ln s) = \frac{2}{3}\left(\frac{f''}{f} - 3\frac{f'}{f} - \frac{f'^2}{f^2}\right)}$$

What have we gained?

**<u>Source does not depends on the solution itself</u>**

# Power-Spectrum
## Generalized Slow-Roll

- Source linked to power-law formula in slow-roll

$$\left. G' \right|_{ks\approx 1} \approx 1 - n_s$$

$$\left. G'' \right|_{ks\approx 1} \approx n_s'$$

$$\left. G''' \right|_{ks\approx 1} \approx n_s''$$

$$\ldots$$

Powerful insights about next-to-leading order corrections

<u>Optimized Slow-Roll</u>

<u>Motohashi & Hu (2015)</u>

# Application of GSR – Part I
## Steps in DBI Inflation

- Too slow for MCMC methods.
- Can only handle case-by-case basis.

# Steps in DBI Inflation

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# Sharp Steps in DBI Inflation

## DBI as a phenomenological model

$$\boxed{\mathcal{L}(X,\phi)=\left[1-\sqrt{1-\frac{2X}{T(\phi)}}\right]T(\phi)-V(\phi)}$$

### General Step

$$F(\phi)=F_0(\phi)\left[1+b\left(\tanh\left(\frac{\phi-\phi_s}{d}\right)-1\right)\right]$$

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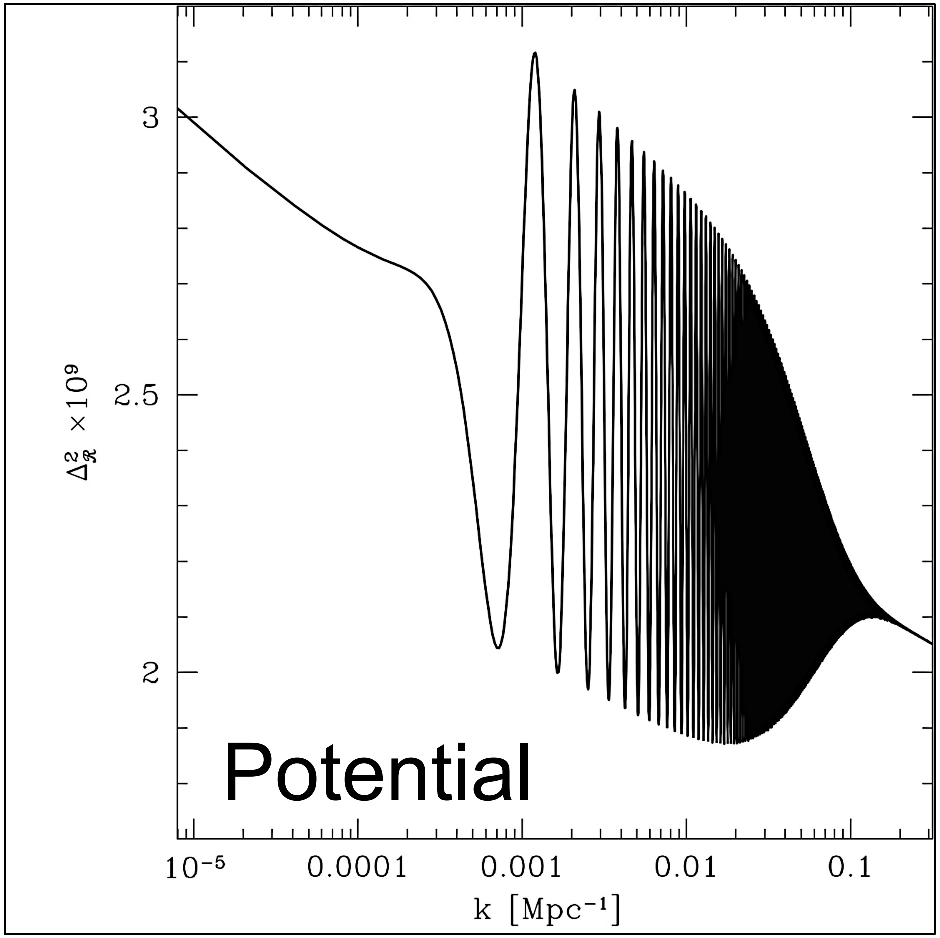
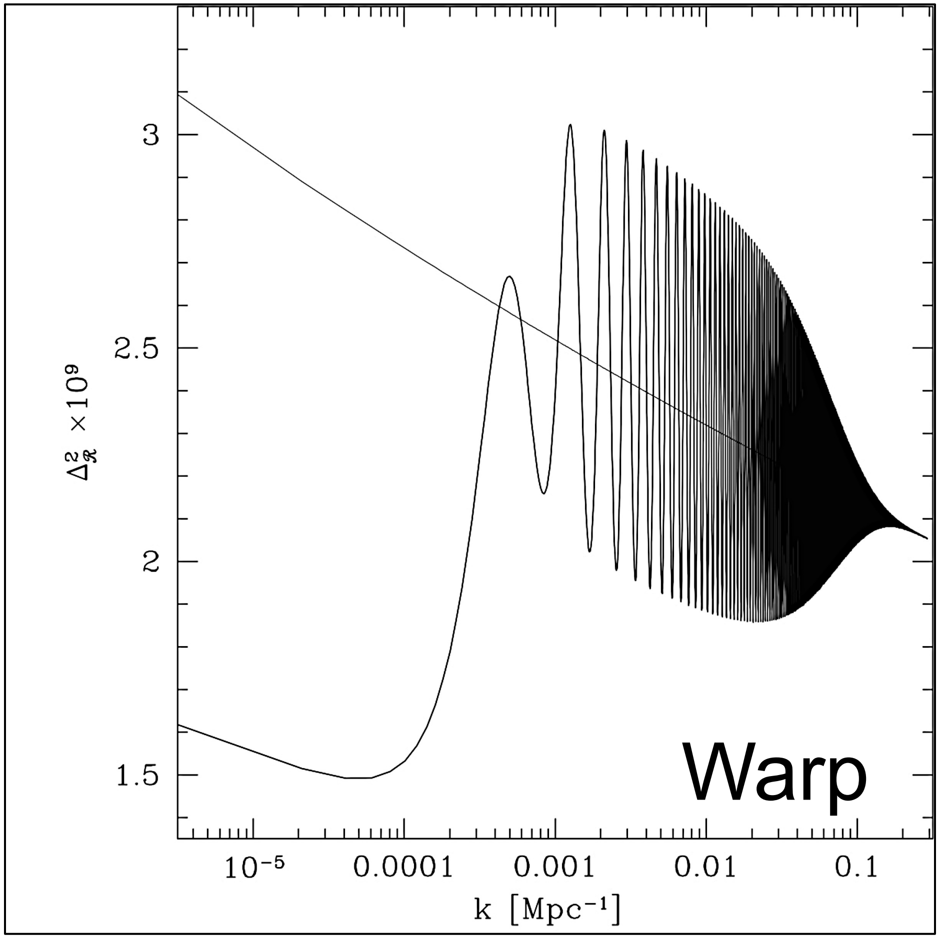
# Sharp Steps in DBI Inflation

## Effects on Curvature Power-Spectrum




Miranda et al (2012), Miranda & Hu (2013), Miranda et al (2014)

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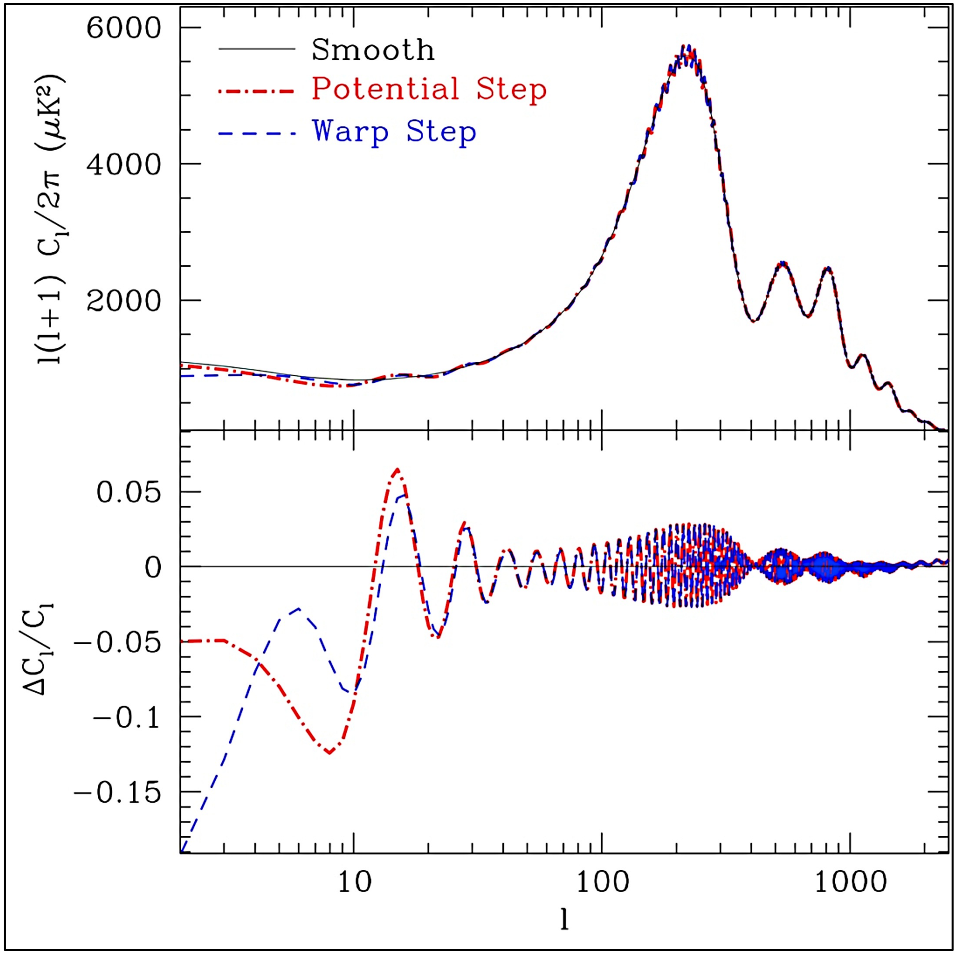
# Sharp Steps in DBI Inflation

## Effects on Temperature Power-Spectrum




**Not so big impact**

**Projection effects**

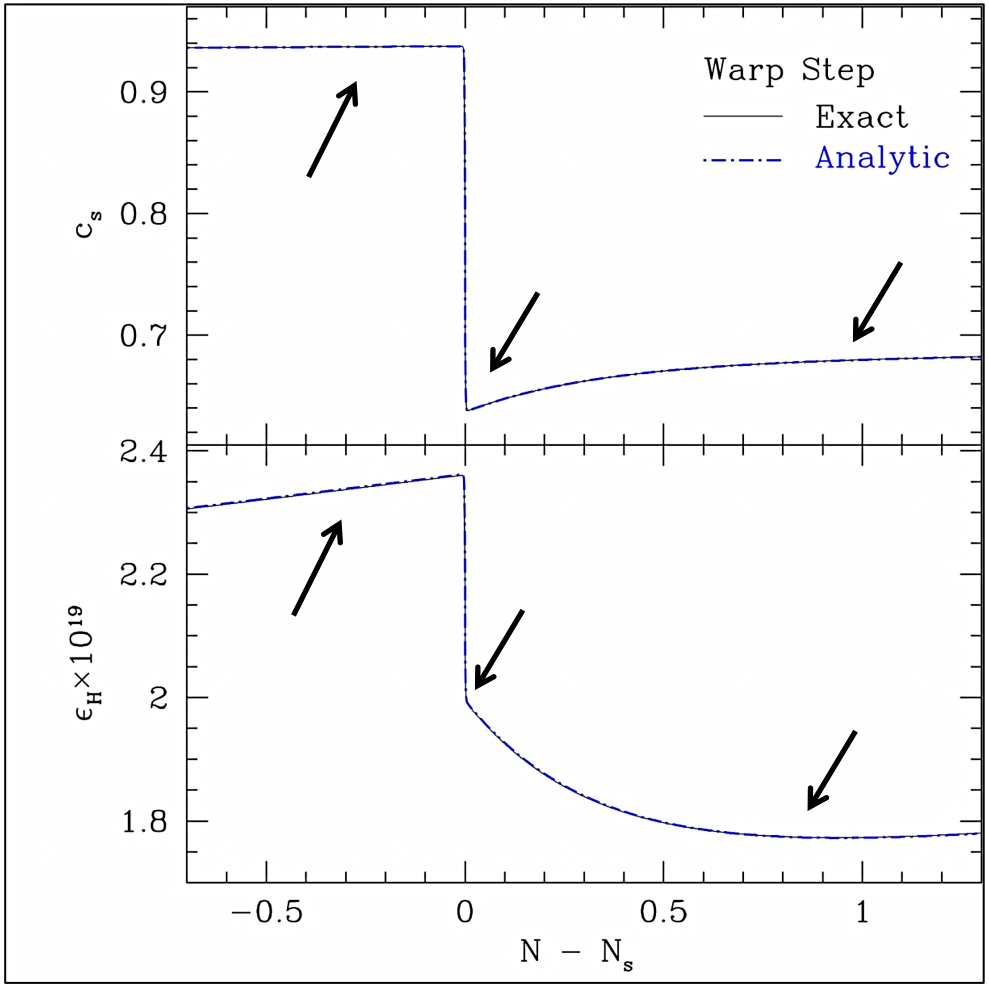
**on high-frequency**

**oscillations!**


Adshead et al (2011), Miranda & Hu (2014),


# Sharp Steps in DBI Inflation

## Analytical Solution




Model Slow-Roll

Evaluate the GSR Source

Approximate the GSR Integral

Adshead et al (2011), Miranda & Hu (2014), Motohashi & Hu (2015)

# Sharp Steps in DBI Inflation

## Analytical Solution

$$\int d\ln s\, W_j(ks)\, G'(\ln s)$$

Model Slow-Roll

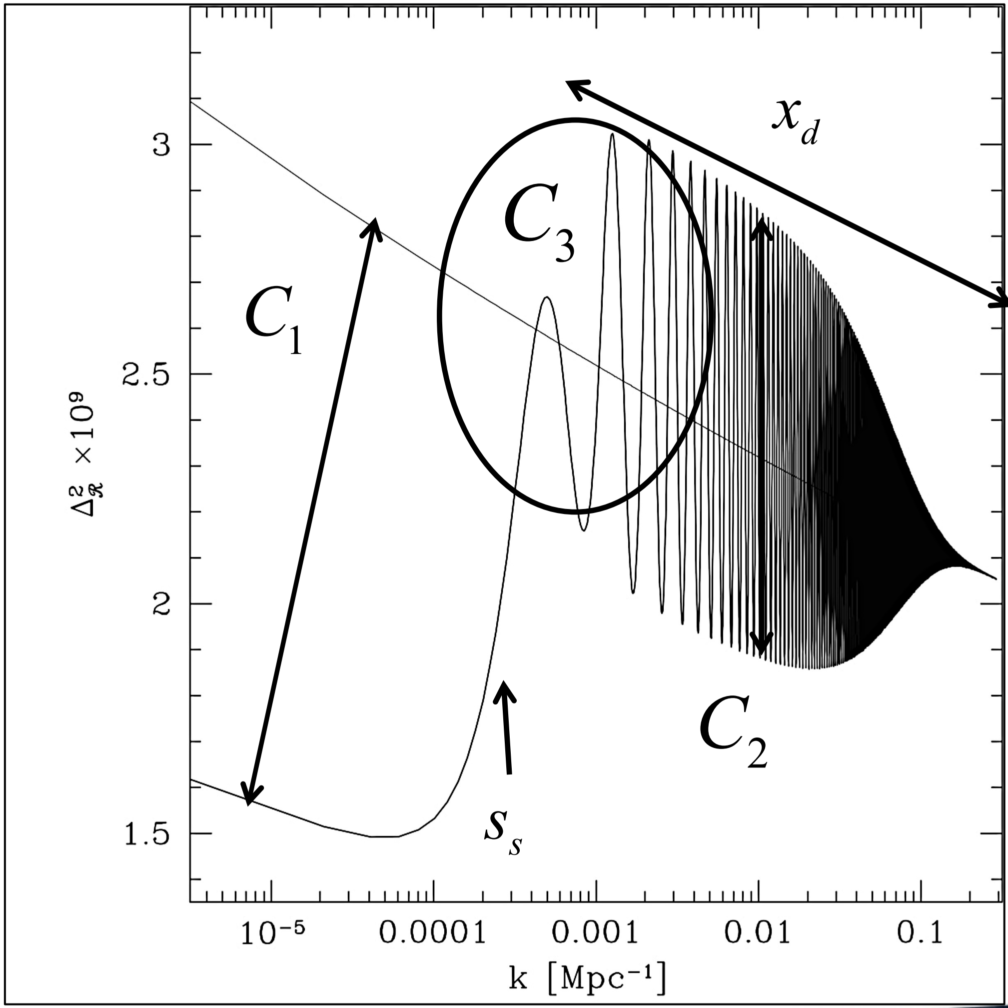
Evaluate the GSR Source

Approximate the GSR Integral

Adshead et al (2011), Miranda & Hu (2014), Motohashi & Hu (2015)

# Sharp Steps in DBI Inflation

## Analytical Solution



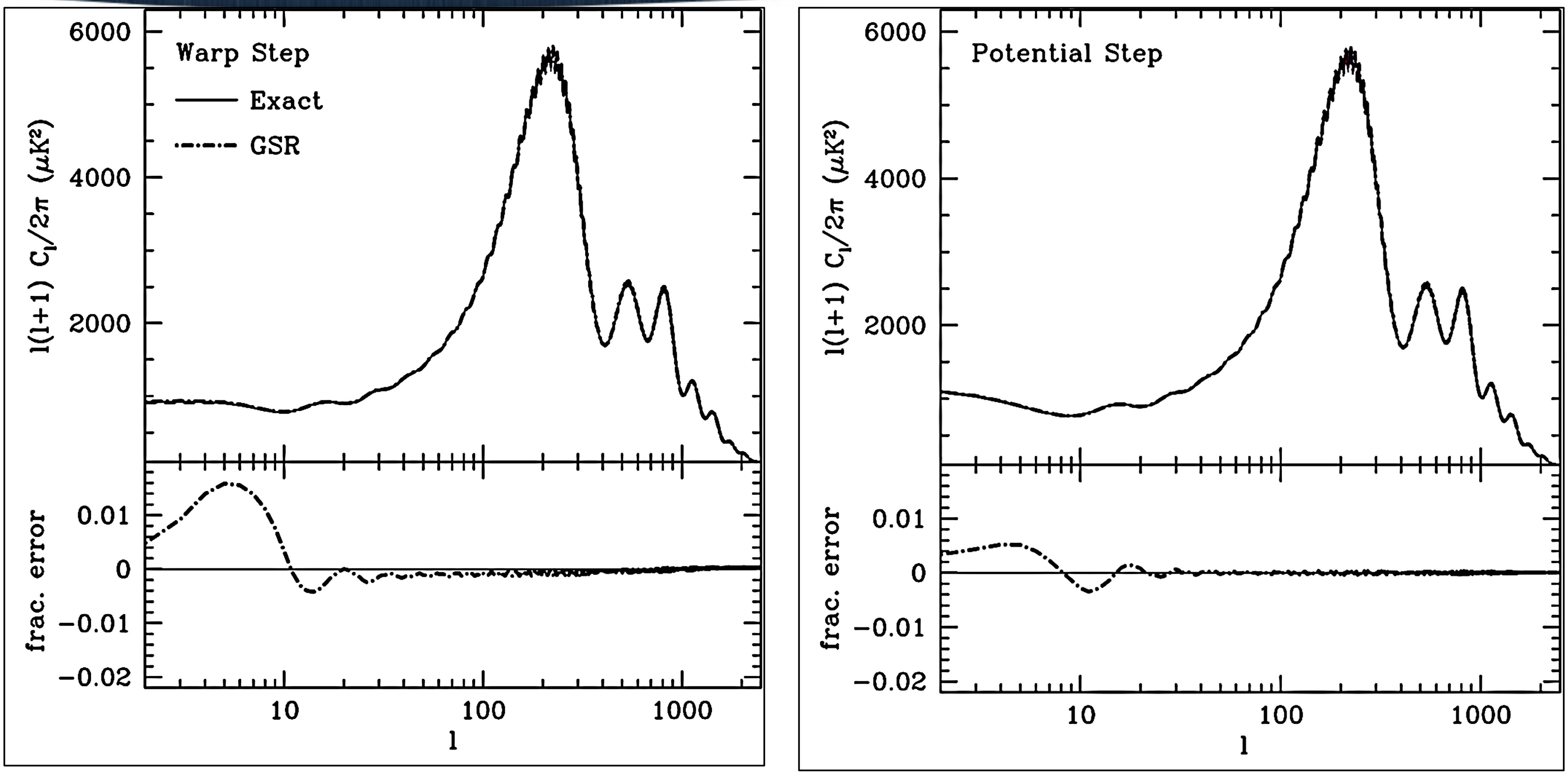

5 parameters

not independent

- Step Amplitude
- Step Position
- Step Width
- Sound speed

# Sharp Steps in DBI Inflation

## Analytical Solution




**Enough Precision for Planck**

Miranda & Hu (2014)

# Sharp Steps in DBI Inflation

## Planck 2015

*9.1. Models*

9.1.1. Step in the inflaton potential

A sudden, step-like feature in the inflaton potential (Adams et al., 2001) or the sound speed (Achúcarro et al., 2011) will lead to a localized oscillatory burst in the scalar primordial power spectrum. A general parameterization describing both a tanh-step in the potential and in the warp term of a DBI model was proposed in Miranda & Hu (2014):

$$\ln \mathcal{P}^{s}_{\mathcal{R}}(k) = \exp\left[\ln \mathcal{P}^{0}_{\mathcal{R}}(k) + \mathcal{I}_0(k) + \ln\left(1 + \mathcal{I}_1^2(k)\right)\right], \quad (71)$$

where the first- and second-order terms are given by

$$\mathcal{I}_0 = \mathcal{A}_s \mathcal{W}_0(k/k_s)\,\mathcal{D}\left(\frac{k/k_s}{x_s}\right), \quad (72)$$

$$\mathcal{I}_1 = \frac{1}{\sqrt{2}}\left[\frac{\pi}{2}(1-n_s) + [\mathcal{A}_s \mathcal{W}_1(k/k_s)+ \mathcal{A}_2 \mathcal{W}_2(k/k_s) + \mathcal{A}_3 \mathcal{W}_3(k/k_s)]\,\mathcal{D}\left(\frac{k/k_s}{x_s}\right)\right], \quad (73)$$

with window functions

$$\mathcal{W}_0(x) = \frac{1}{2x^4}\left[\left(18x - 6x^3\right)\cos 2x + \left(15x^2 - 9\right)\sin 2x\right], \quad (74)$$

$$\mathcal{W}_1(x) = -\frac{1}{x^4}\left\{3(x\cos x - \sin x)\left[3x\cos x + \left(2x^2-3\right)\sin x\right]\right\}, \quad (75)$$

$$\mathcal{W}_2(x) = \frac{3}{x^3}(\sin x - x\cos x)^2, \quad (76)$$

$$\mathcal{W}_3(x) = -\frac{1}{x^3}\left[3 + 2x^2 - \left(3 - 4x^2\right)\cos(2x) - 6x\sin(2x)\right], \quad (77)$$

Planck has followed our proposed approach

- Assumed $\boxed{c_s = 1}$
- Constrains $\boxed{\Delta\chi^2 < 10}$

Final word will be given by <u>polarization</u>

# Application of GSR – Part II
## Anomalies in the temperature spectrum

- Too slow for MCMC methods.
- Can only handle case-by-case basis.

# Anomalies in the temperature spectrum

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# Power-Spectrum
## Generalized Slow-Roll

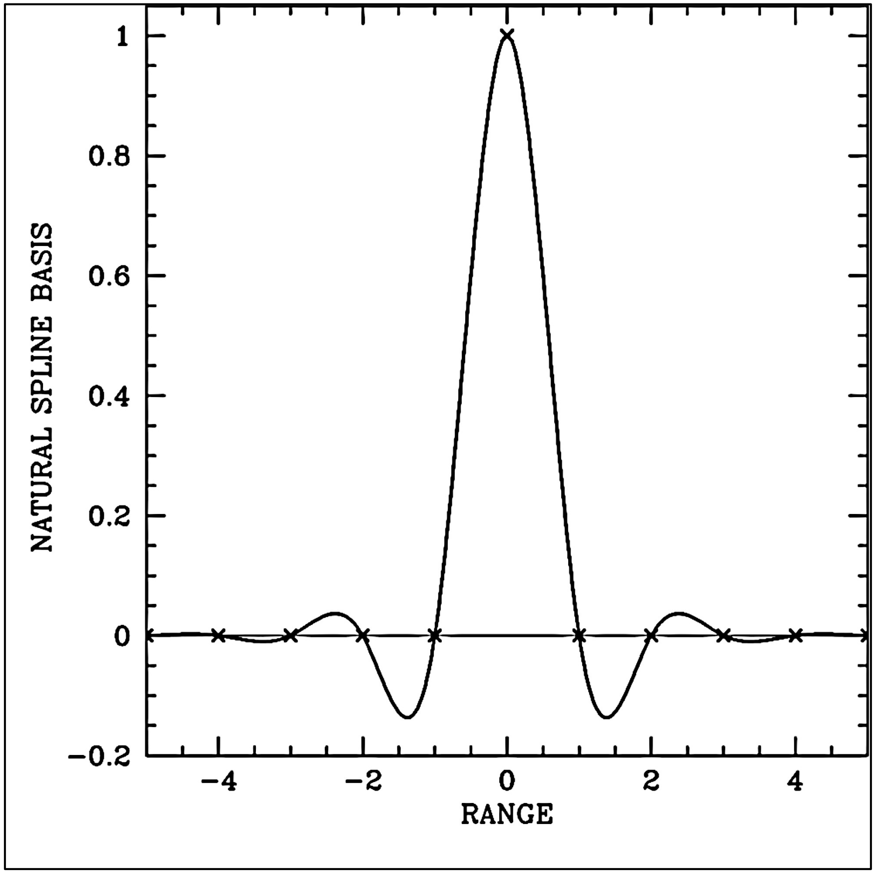
$$\boxed{\ln \Delta_{\mathcal{R}}^2 = I_0(k) + \ln[1 + I_1^2(k)]}$$

$$\boxed{I_j(k) \propto \int d\ln s\, W_j(ks)\, G'(\ln s)}$$

All phenomenology encoded in a single source

Dvorkin & Hu (2010, 2011), Miranda et al (2014)

# Model Independent Analysis
## Choice of basis

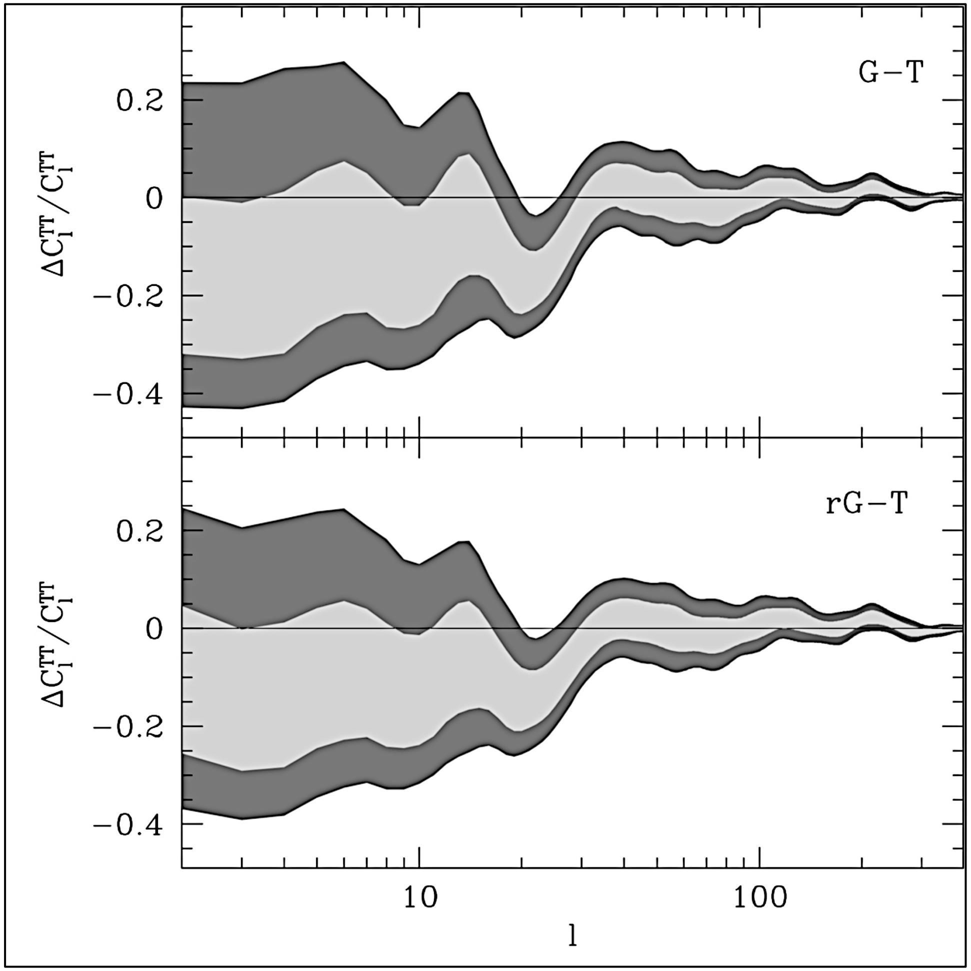



$$G'(\ln s) - (1 - n_s) = \sum_i B_i(\ln s) w_i$$

- Natural Spline Basis
- Range (< second peak)
- Number: 20 components

Miranda et al (2014)

# Model Independent Analysis
# What is the fit?

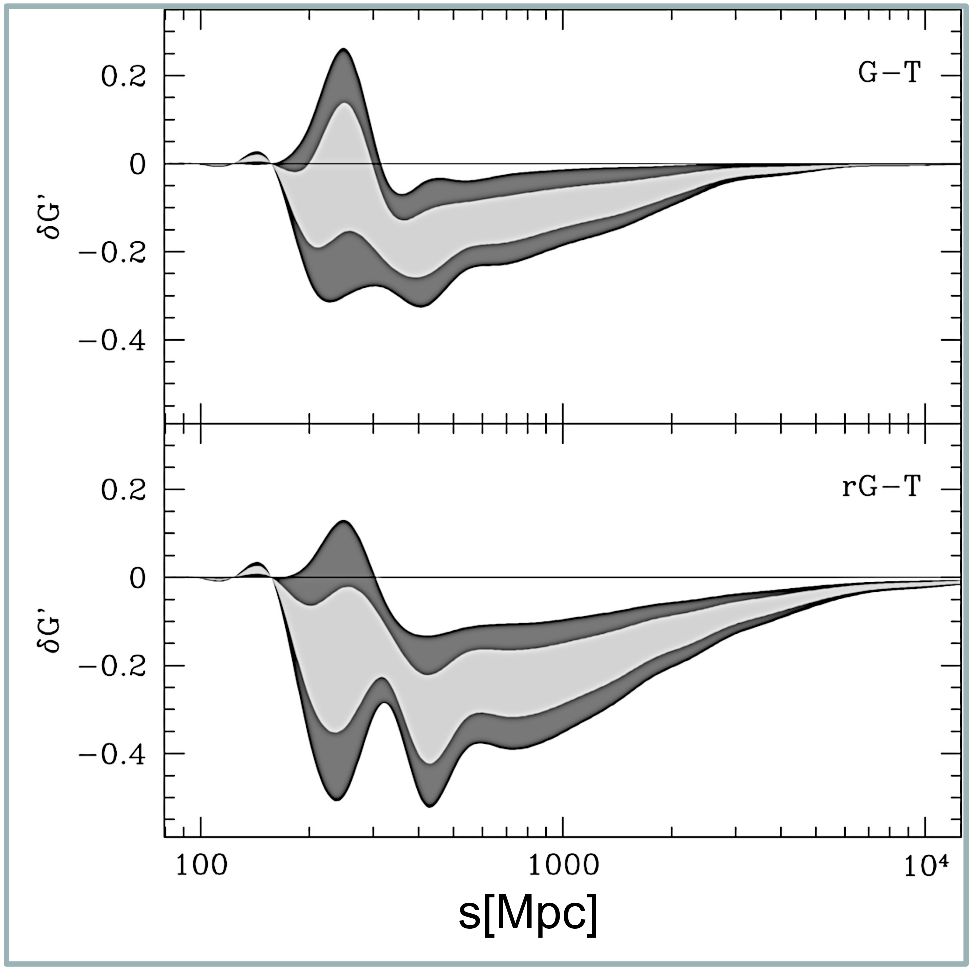



Planck, r=0 (no tensors)

Planck, free-r

Miranda et al (2014)

# Model Independent Analysis
# What is the reconstructed source?

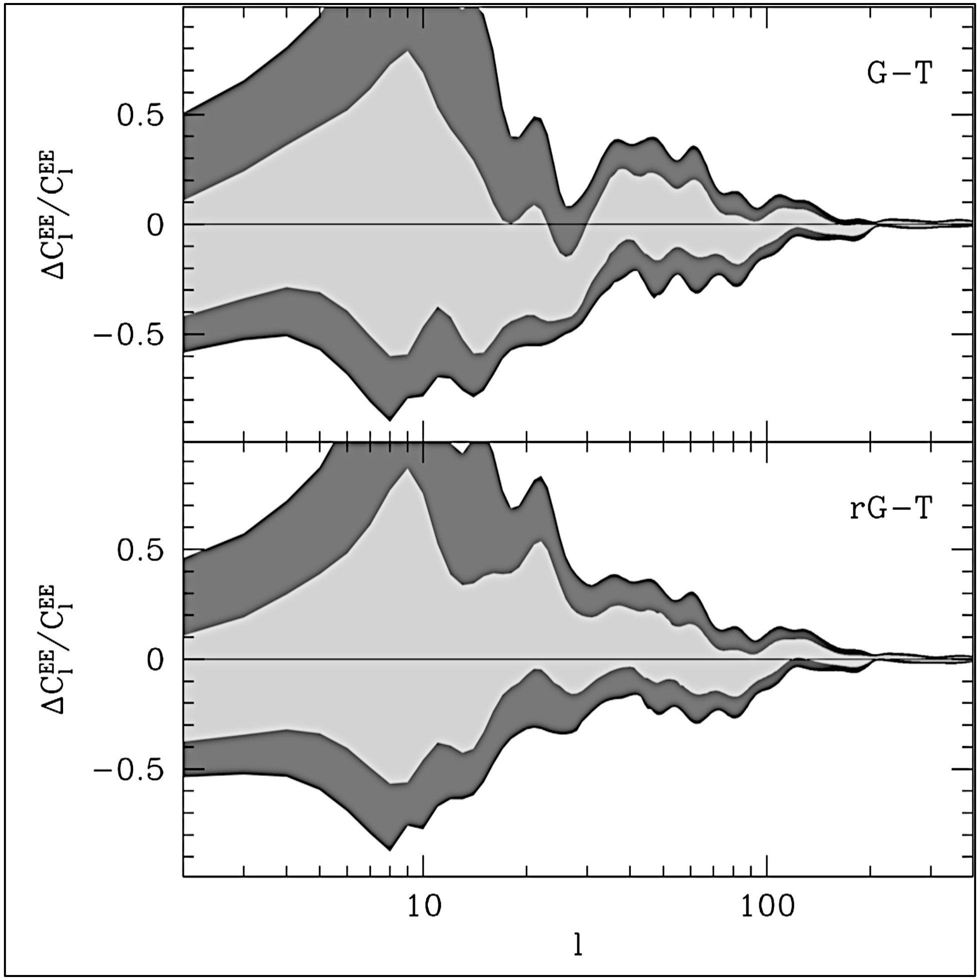



Planck, r=0 (no tensors)

Planck, free-r

Miranda et al (2014)

# Model Independent Analysis
# What is the prediction?




Planck, r=0 (no tensors)

Planck, free-r

Miranda et al (2014)

# Conclusions

- Single-field inflation provides phenomenology much richer than what can be described by a simple power-law.
- GSR provides a framework for constructing accurate analytical solutions in models that violate the slow-roll approximation.
- GSR provides a framework for model-independent constrains in single-field inflation.

Thank You