

LENSING OF POINT SOURCES

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Outline

I. **Weak**-lensing of supernovae Ia

Interested in the full sample (strong lensing is rare)

II. **Strong**-lensing of quasars

Interested in the (small) strongly lensed sample

Weak-lensing basics

magnification

$$\mu = \frac{1}{(1 - \kappa)^2 - \gamma^2}$$

shift in
dist. modulus

$$\Delta m = 5 \log_{10} \mu^{-1/2} \simeq -\frac{5}{\ln 10} \kappa$$

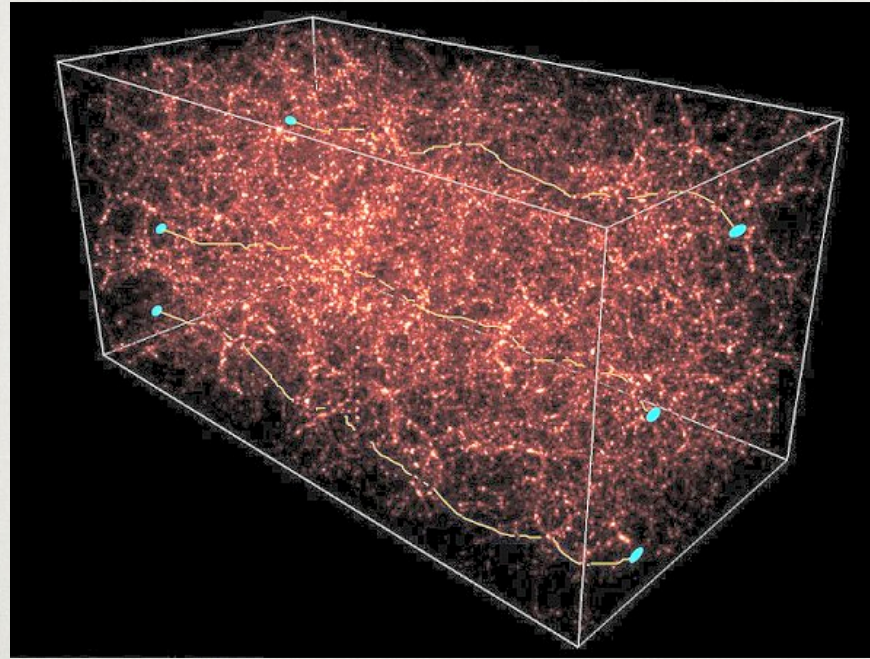
weak-lensing
regime ($k \ll 1$)

The **effective convergence** is a **weighted line-of-sight integral** of the **matter contrast**

$$\kappa(z) = \rho_{MC} \int_0^{r_s(z)} dr G(r, r_s(z)) \delta_M(r, t(r))$$


$$G(r, r_s) = \frac{4\pi G}{c^2} \frac{f_K(r) f_K(r_s - r)}{f_K(r_s)} \frac{1}{a}$$

Weak-lensing basics



$$\langle \delta_M \rangle = 0 \quad \xrightarrow{\Delta m = -\frac{5}{\ln 10} \rho_{MC} \int dr G \delta_M} \quad \langle \Delta m \rangle = 0 \quad \text{no lensing on average}$$

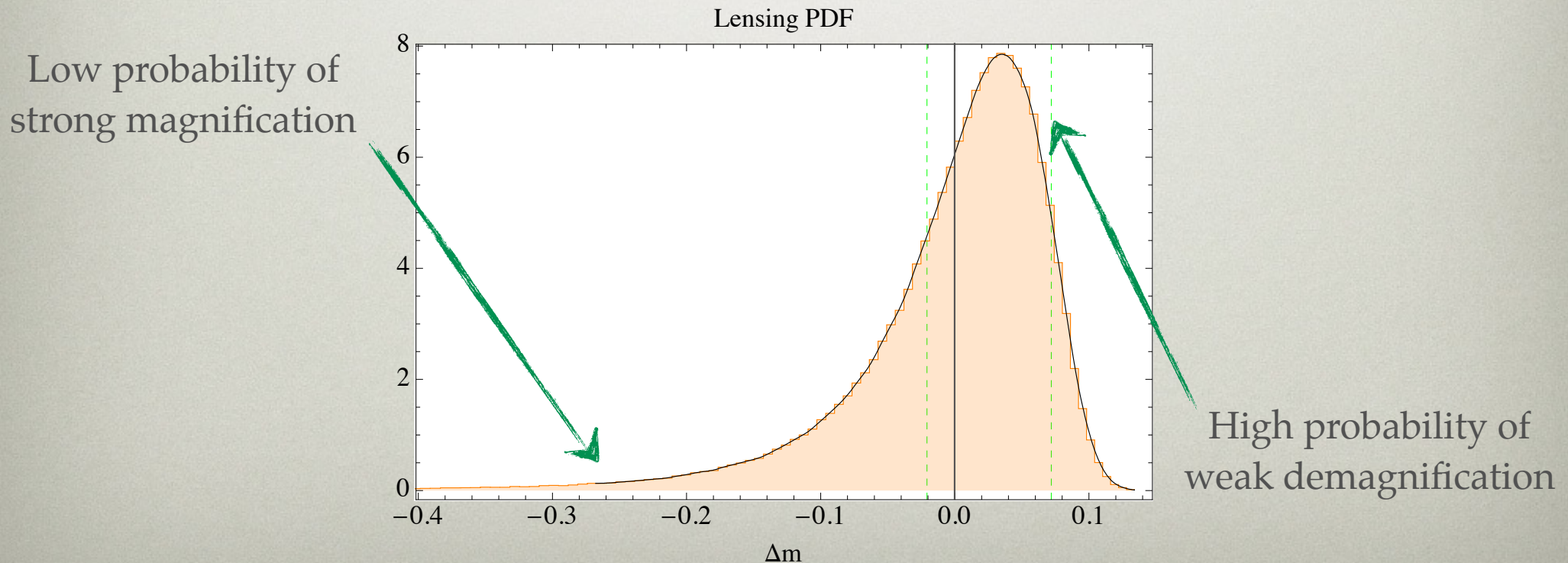
This is true also within full nonlinear lensing
because of photon flux conservation[§]

[§]A few words of caution may apply, use appropriately.

So, what's interesting about lensing?

The lensing distribution is a proxy for the distribution of the matter contrast

$$\Delta m = -\frac{5}{\ln 10} \rho_{MC} \int dr G \delta_M$$



sGL method & turboGL code

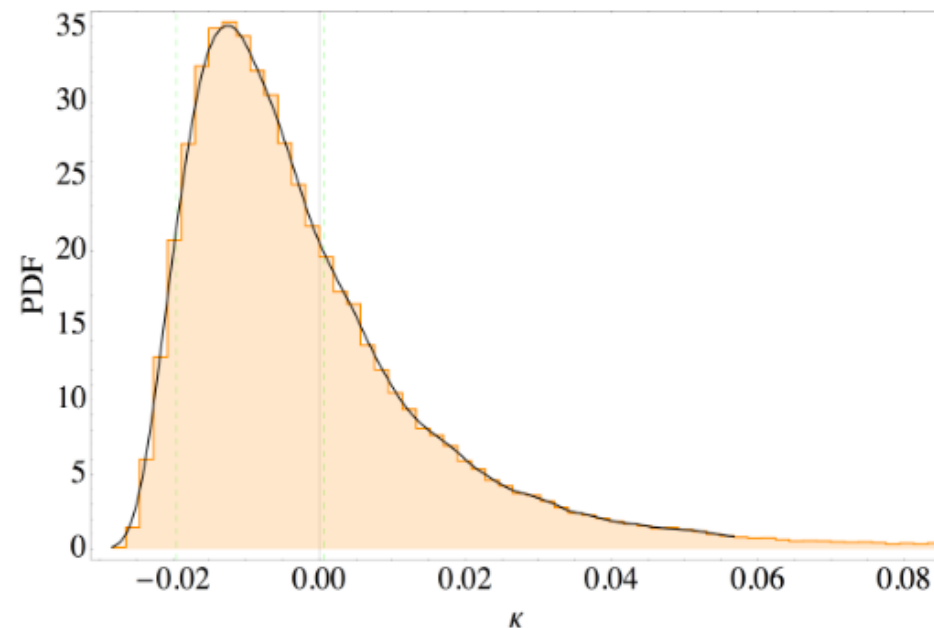
stochastic Gravitational Lensing method

Kainulainen & Marra,
0909.0822, 1011.0732, 1101.4143

Code available^s at turboGL.org



Welcome to the turboGL website!



turboGL is a fast Mathematica code based on the stochastic approach to cumulative weak lensing (the sGL method extended/revised in Ref. [4]). turboGL can easily compute the lensing PDF relative to arbitrary halo mass number of observations, halo profiles and evolutions. Check the [version history](#) for the features included in the current version. To be informed of a new version, please send an [email](#). Do not hesitate to contact me if you have any questions.

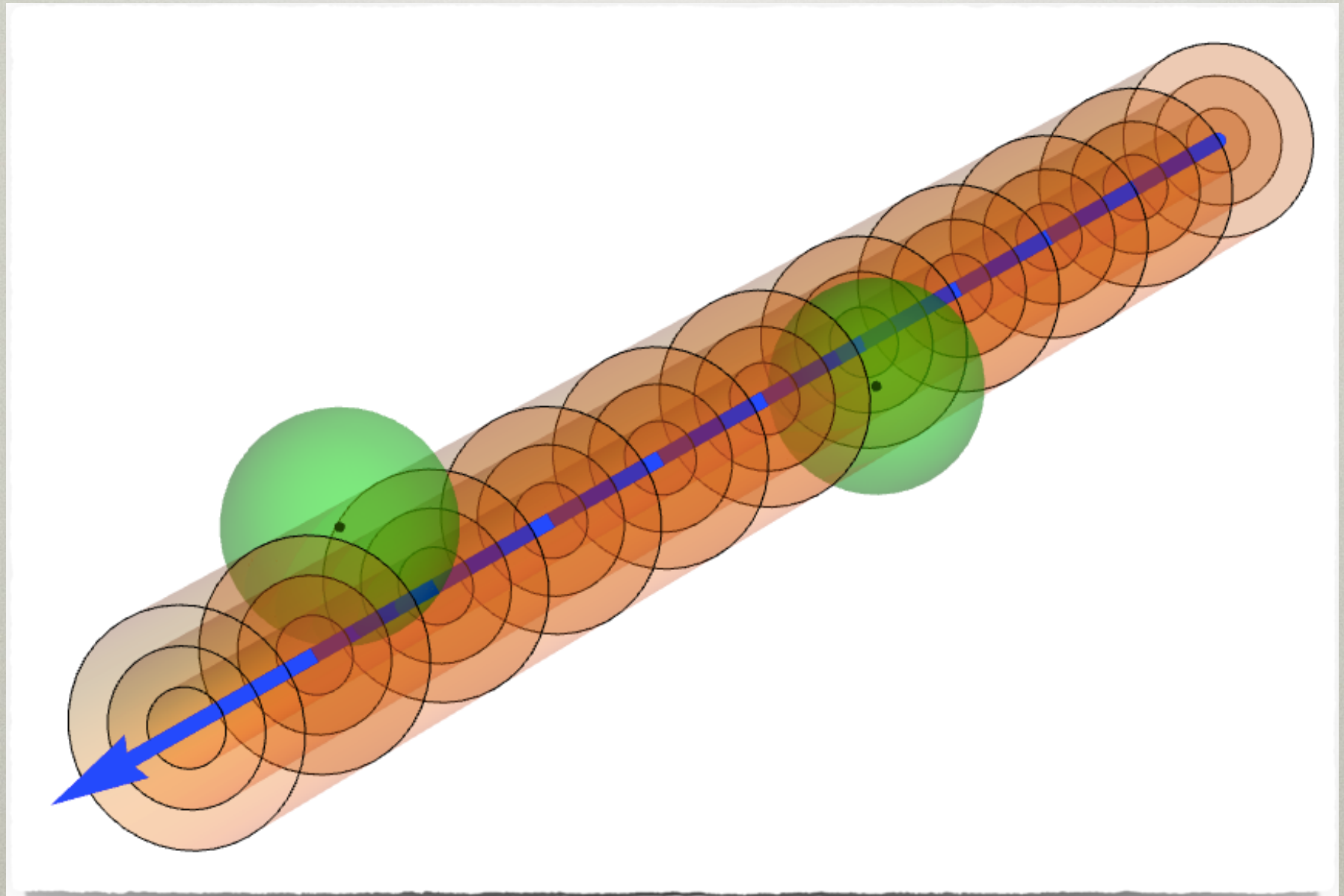
[Valerio Marra](#)

Visits: 1454 W3C

PS: if interested in void models, visit the [LLTB homepage](#)

turboGL 2.0 - 30 Apr 2013 - [download](#)

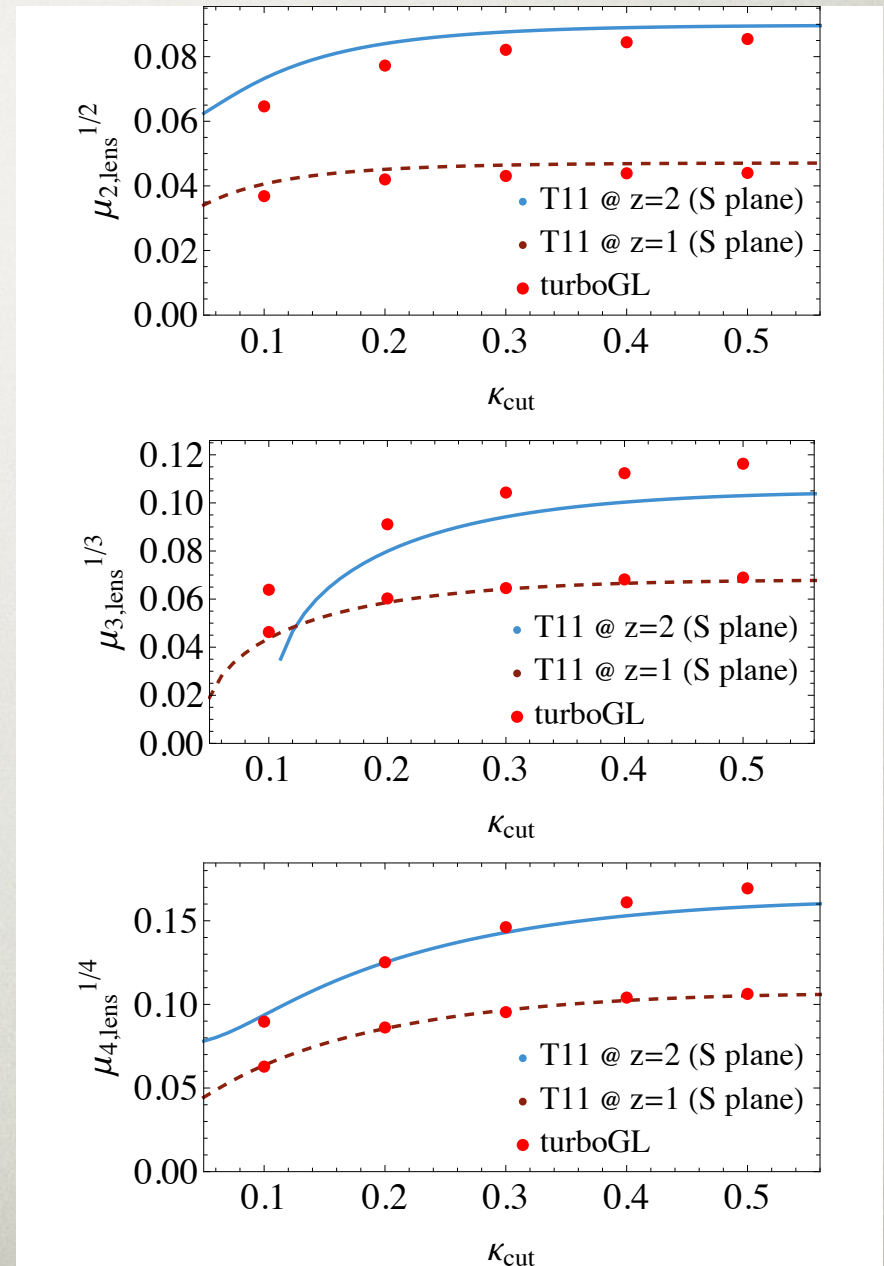
sGL method & turboGL code



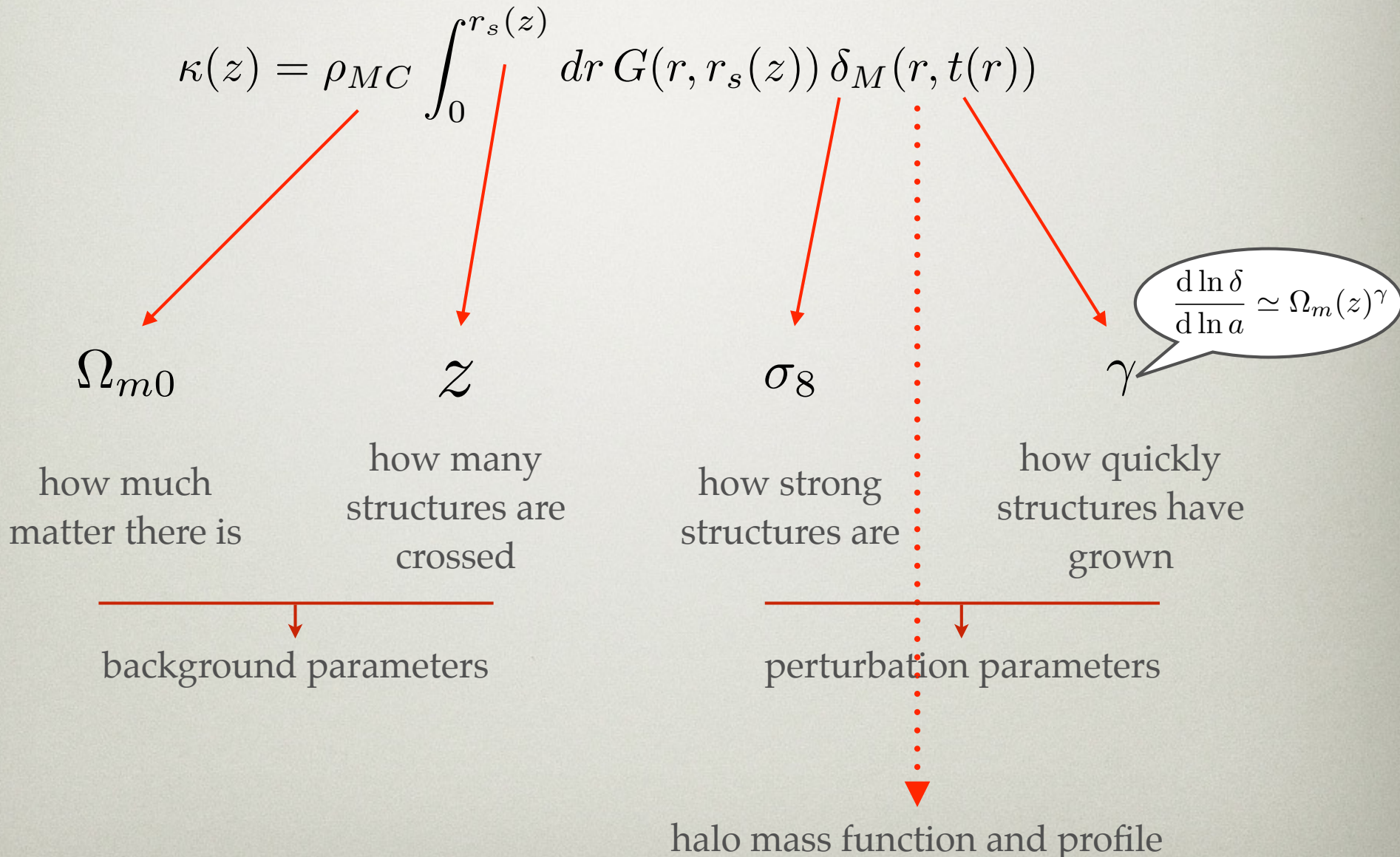
Focusing on the moments

The non-gaussian information contained in the lensing PDF can be condensed into its central moments:

$$\mu_n \equiv \langle (X - \langle X \rangle)^n \rangle$$

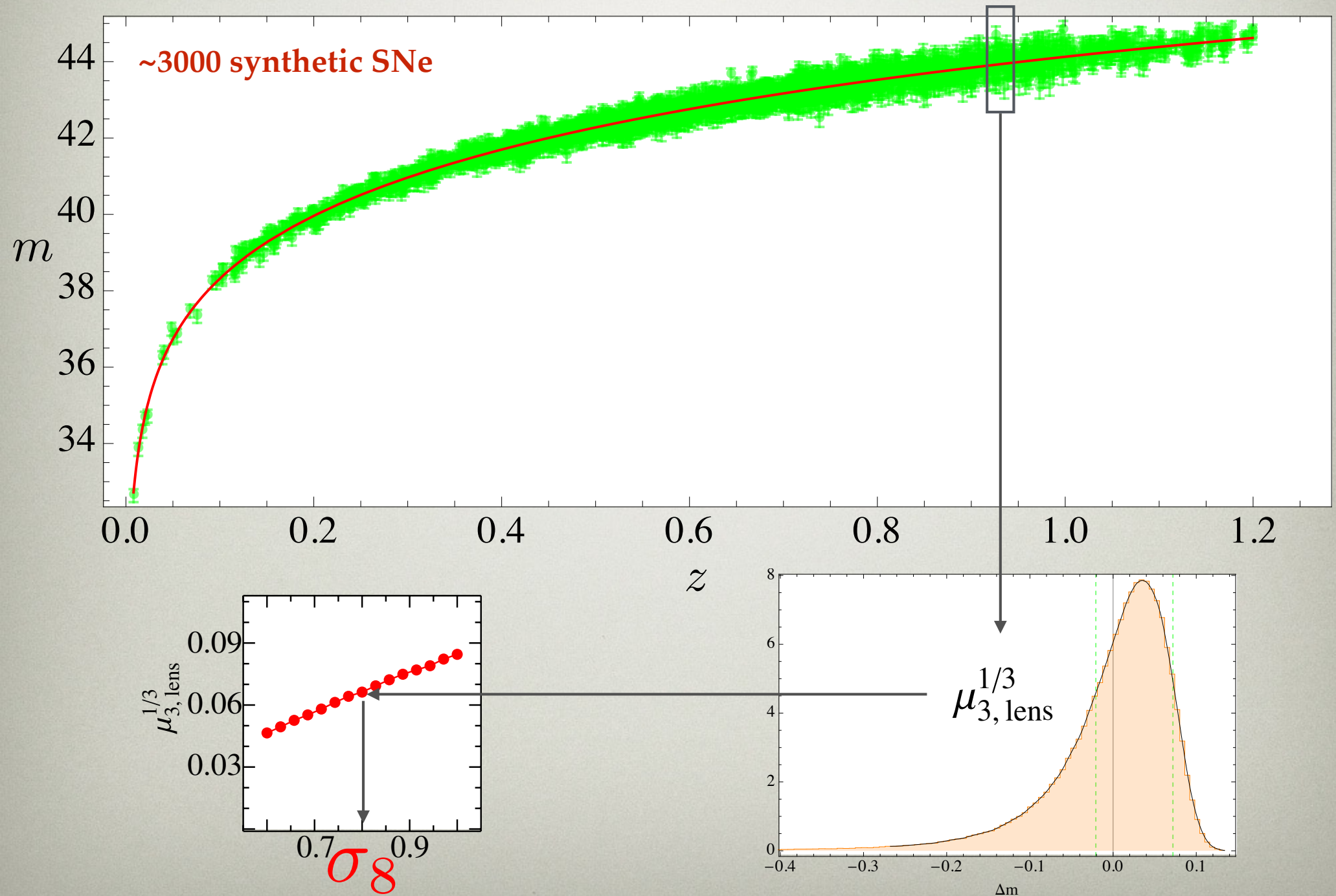


Lensing (strongly) depends[§] on...



[§]Fits available, tutti frutti: Marra et al 1304.7689, Quartin et al 1307.1155, Amendola et al 1412.3703.

Signal from noise



Method of the Moments

a χ^2 approach where
one compares measured
to predicted moments

$$L_{\text{MeMo}}(\text{Data}|\Theta_{\text{cosmo}}) = \exp \left(-\frac{1}{2} \sum_j^{\text{bins}} \chi_j^2 \right)$$

$$\chi_j^2 = (\boldsymbol{\mu} - \boldsymbol{\mu}_{\text{data}})^t \boldsymbol{\Sigma}_j^{-1} (\boldsymbol{\mu} - \boldsymbol{\mu}_{\text{data}}),$$

$$\boldsymbol{\mu} = \{\mu'_1, \mu_2, \mu_3, \mu_4\},$$

convolution of lensing PDF
with intrinsic SN mag PDF

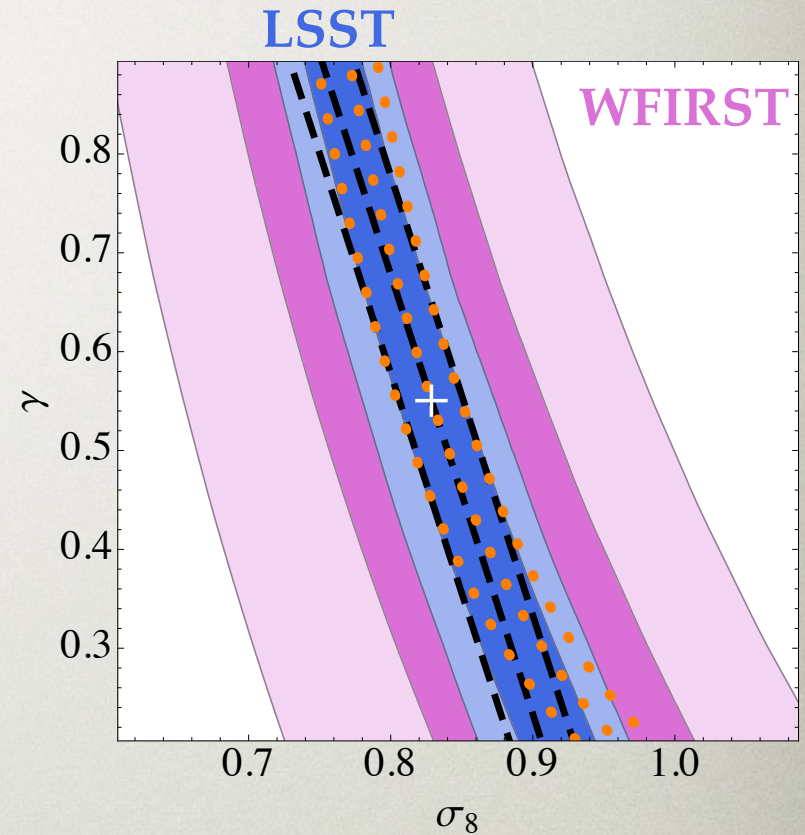
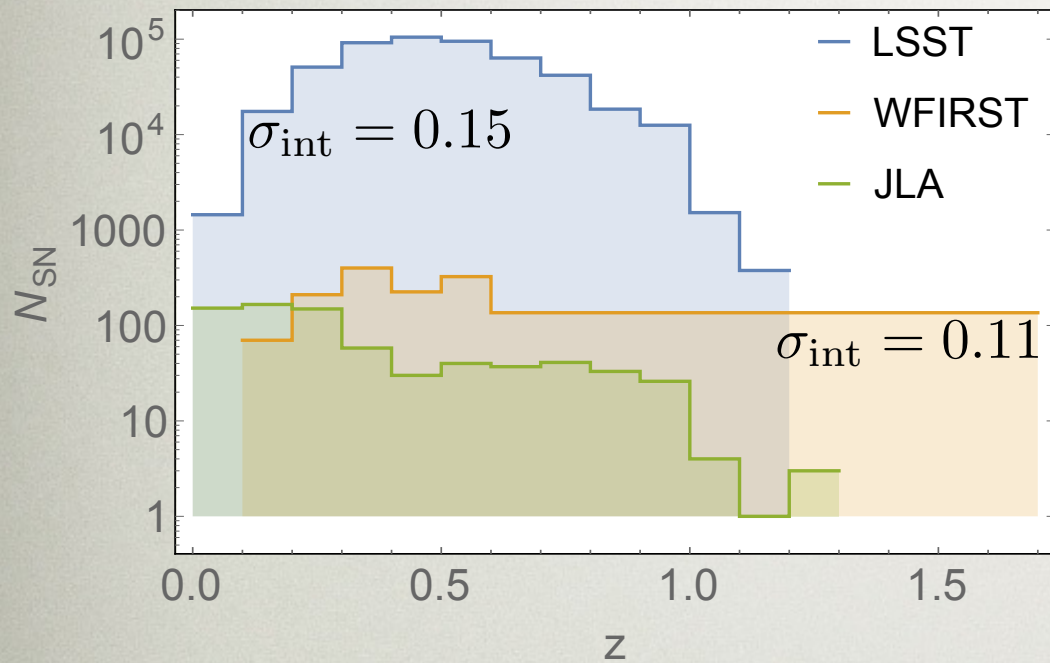
$$\mu_2 \equiv \sigma_{\text{tot}}^2 = \sigma_{\text{lens}}^2 + \sigma_{\text{int}}^2,$$

$$\mu_3 = \mu_{3,\text{lens}} + \mu_{3,\text{int}},$$

$$\mu_4 = \mu_{4,\text{lens}} + 6 \sigma_{\text{lens}}^2 \sigma_{\text{int}}^2 + 3 \sigma_{\text{int}}^4 + \mu_{4,\text{int}}$$

the intrinsic SN PDF is assumed to be
redshift-independent: the posterior is
marginalized over the intrinsic moments

Constraining the growth of perturbations



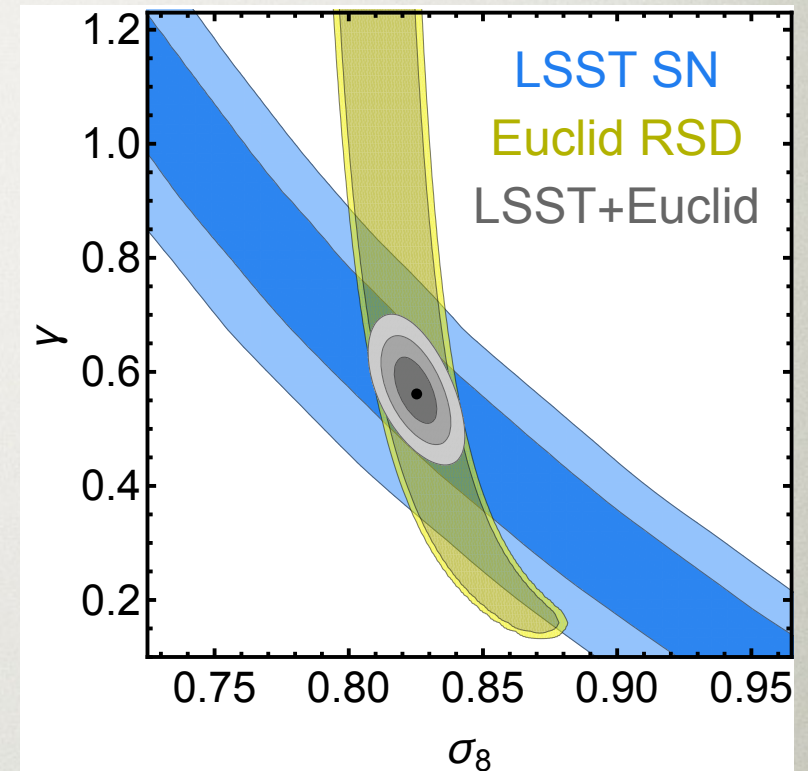
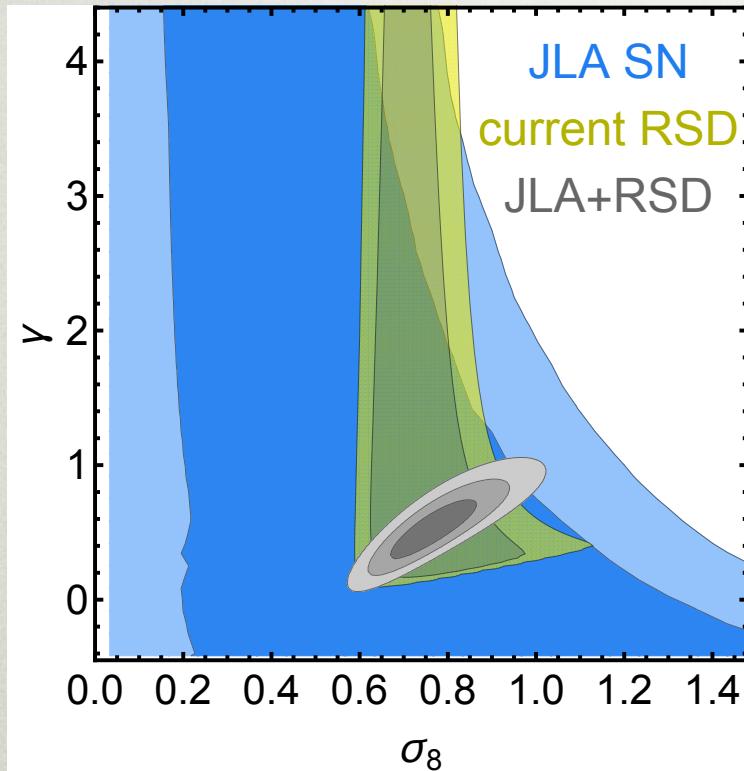
Degeneracy between growth index and amplitude of the power spectrum

$$\gamma(\sigma_8/0.83)^{6.7} = 0.55 \pm 0.1$$

$$\frac{d \ln \delta}{d \ln a} \simeq \Omega_m(z)^\gamma$$

Redshift distortions + SN lensing

RSD:
2dFGS
6dFGS
LRG
BOSS
CMASS
WiggleZ
VIPERS



growth rate
likelihood

$$L_{\text{gr}} = L_{\text{gr},0} \exp \left[-\frac{1}{2} (d_i - t_i) C_{ij}^{-1} (d_j - t_j) \right]$$

$$d = f(z) \sigma_8(z)$$

Strong lensing

Lens equation:

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \frac{D_{\text{ds}}}{D_{\text{s}}} \hat{\boldsymbol{\alpha}}(D_{\text{d}} \boldsymbol{\theta}) \equiv \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$$

(Scaled) deflection angle:

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{1}{\pi} \int_{\mathbb{R}^2} d^2\theta' \kappa(\boldsymbol{\theta}') \frac{\boldsymbol{\theta} - \boldsymbol{\theta}'}{|\boldsymbol{\theta} - \boldsymbol{\theta}'|^2}$$

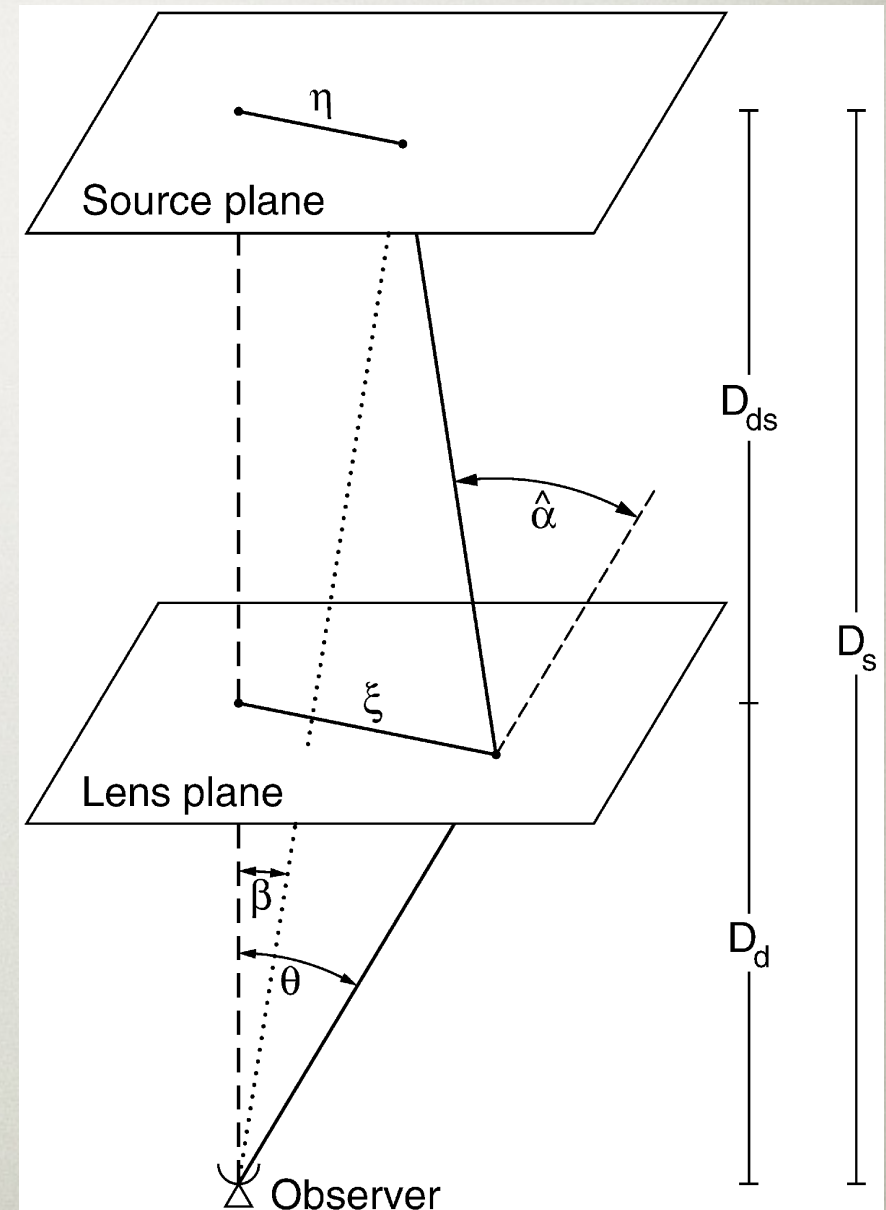
$$\kappa(\boldsymbol{\theta}) := \frac{\Sigma(D_{\text{d}} \boldsymbol{\theta})}{\Sigma_{\text{cr}}} \quad \text{with} \quad \Sigma_{\text{cr}} = \frac{c^2}{4\pi G} \frac{D_{\text{s}}}{D_{\text{d}} D_{\text{ds}}}$$

halo surface
density

halo profile
parameters

cosmological
distances

background
parameters



Splitting angles statistics

The dark matter profile of halos does not strongly depend on cosmology. Rather, it is substantially affected by baryons.

The statistics of double images can be used to study how observations constrain the DM and baryon profiles.

For example, J-PAS (1403.5237) is expected to observe around 2.5×10^6 quasars up to $z=2.5$, and 2.1×10^4 quasars, in the range $4 < z < 7$.

GNFW profile	$\rho(r) = \frac{\rho_s r_s^3}{r^\alpha (r + r_s)^{3-\alpha}}$	$\alpha = 1$	NFW profile
		$\alpha = 2$	SIS-NFW profile

sGL is being extended to the case of strong lensing.

In collaboration with Kimmo Kainulainen and Mikko Pääkkönen

THANKS