VI Workshop Challenges in New Physics in Space

From Quantum to Classical Instability in Relativistic Stars

André G. S. Landulfo (Center for Natural and Human Sciences/ UFABC)

In collaboration with W. Lima, G. Mastsas, and D. Vanzella

Awaking the vacuum

Awaking the vacuum

- It has been shown that relativistic stars may become unstable due to quantum effects: The vacuum awaking effect [W. Lima, D. Vanzella, Phys. Rev. Lett. 104, 161102 (2010), W. Lima, G. Matsas, D. Vanzella, Phys. Rev. Lett. 105, 151102 (2010), W. Lima, R. Mendes, G. Matsas, D. Vanzella, Phys. Rev. D 87, 104039 (2013)]

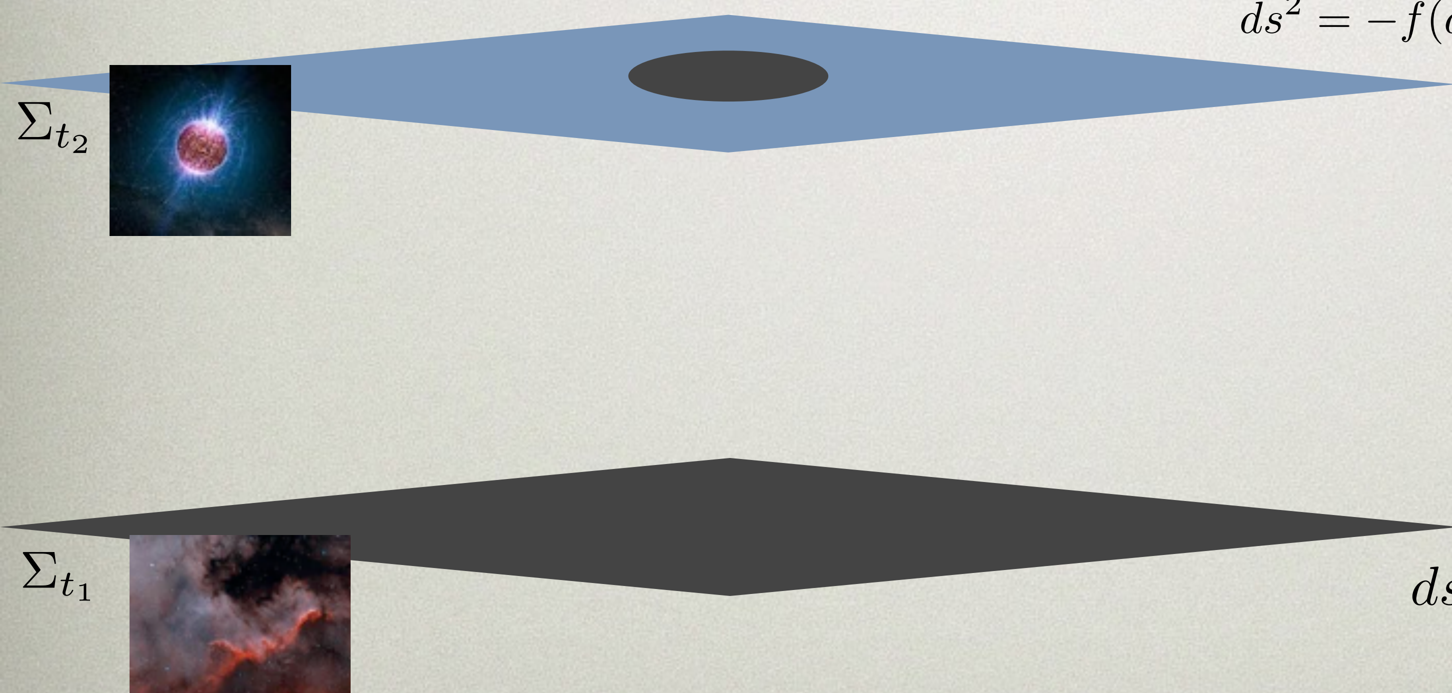
Awaking the vacuum

Awaking the vacuum

Σ_{t_1}

$$ds^2 = -dt^2 + d\mathbf{x}^2$$

Awaking the vacuum



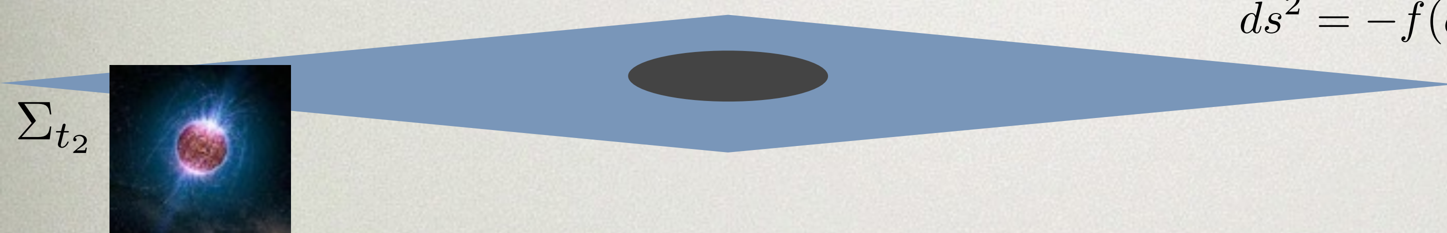
$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

$$ds^2 = -dt^2 + d\mathbf{x}^2$$

Awaking the vacuum

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_{t_2}



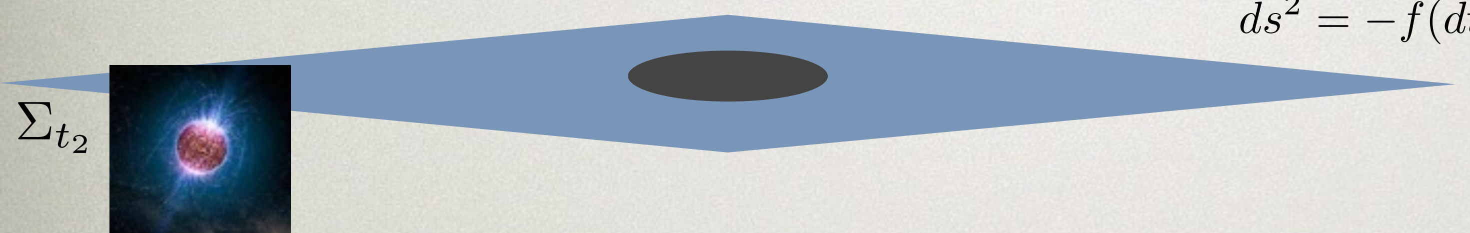
$$(-\nabla_a \nabla^a + \xi R)\Phi = 0$$

Σ_{t_1}

$$ds^2 = -dt^2 + d\mathbf{x}^2$$

Awaking the vacuum

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



$$(-\nabla_a \nabla^a + \xi R)\Phi = 0$$

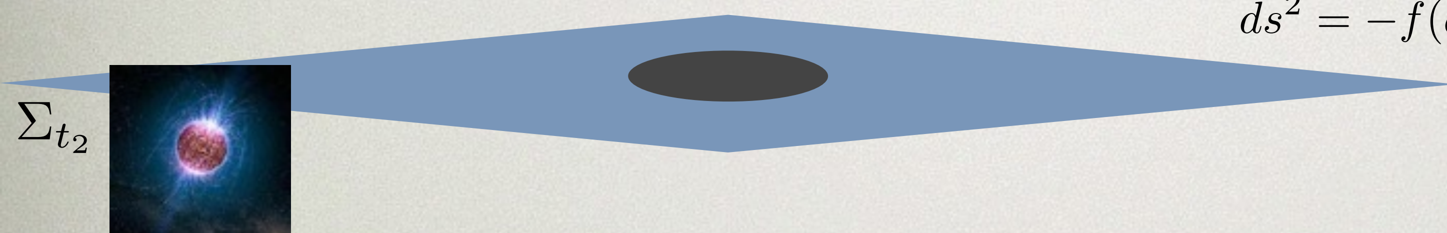
$$ds^2 = -dt^2 + d\mathbf{x}^2$$

$$|0_{\text{in}}\rangle$$

Awaking the vacuum

$$\langle \hat{\Phi}^2 \rangle_{\rm in} \sim \frac{\kappa e^{2\bar{\Omega} t}}{2\bar{\Omega}} \left(\frac{\bar{F}(\mathbf{x})}{f_{\rm int}(\mathbf{x})} \right)^2 [1 + \mathcal{O}(e^{-\epsilon t})]$$

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta \, d\varphi^2)$$



Σ_{t_2}

$$(-\nabla_a \nabla^a + \xi R)\Phi = 0$$

Σ_{t_1}

$$ds^2 = -dt^2 + d\mathbf{x}^2$$

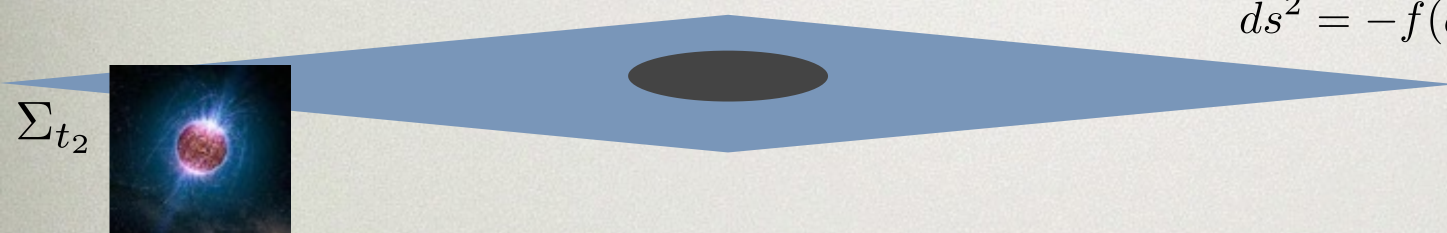
$$|0_{\rm in}\rangle$$

Awaking the vacuum

$$\langle\hat{\Phi}^2\rangle_{\rm in} \sim \frac{\kappa e^{2\bar{\Omega}t}}{2\bar{\Omega}}\left(\frac{\bar{F}(\mathbf{x})}{f_{\rm int}(\mathbf{x})}\right)^2[1+\mathcal{O}(e^{-\epsilon t})]$$

$$\langle\hat{\rho}\rangle_{\rm in} \sim \kappa\frac{\bar{\Omega}e^{2\bar{\Omega}t}}{16\pi\sqrt{f}}\left\{\frac{1-4\xi}{2r^2}\frac{d}{d\chi}\left(r^2\frac{d}{d\chi}\left(\frac{F_{\bar{\Omega}0}}{\bar{\Omega}r}\right)^2\right) + \frac{\xi}{\bar{\Omega}^2r^2}\frac{d}{d\chi}\left(\frac{{F_{\bar{\Omega}0}}^2}{f}\frac{df}{d\chi}\right)\right\}[1+\mathcal{O}(e^{-\epsilon t})].$$

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



$$(-\nabla_a\nabla^a + \xi R)\Phi = 0$$

$$ds^2 = -dt^2 + d\mathbf{x}^2$$

$$|0_{\rm in}\rangle$$

Vacuum dominance

Vacuum dominance

- Vacuum energy density

Vacuum dominance

- Vacuum energy density

$$\frac{\rho_{\rm v}}{\rho} \sim \left(\frac{\ell_{\rm p}}{R}\right)^2 \times \exp\left(2t/R\right)$$

Vacuum dominance

- Vacuum energy density

$$\frac{\rho_{\rm v}}{\rho} \sim \left(\frac{\ell_{\rm p}}{R}\right)^2 \times \exp\left(2t/R\right) \longrightarrow t_{\rm br} \sim \mathsf{R} \ln(\mathsf{R}/\ell_{\rm P})$$

Vacuum dominance

- Vacuum energy density

$$\frac{\rho_{\mathrm{v}}}{\rho} \sim \left(\frac{\ell_{\mathrm{p}}}{R}\right)^2 \times \exp\left(2t/R\right) \longrightarrow t_{\mathrm{br}} \sim \mathsf{R}\ln(\mathsf{R}/\ell_{\mathrm{P}}) \qquad \rho_{\mathrm{v}} \sim \rho$$

Vacuum dominance

- Vacuum energy density

$$\frac{\rho_{\rm v}}{\rho} \sim \left(\frac{\ell_{\rm p}}{R}\right)^2 \times \exp\left(2t/R\right) \longrightarrow t_{\rm br} \sim \mathsf{R} \ln(\mathsf{R}/\ell_{\rm P}) \qquad \rho_{\rm v} \sim \rho$$

- In a few milliseconds the vacuum take control over the energy density of the compact object

Vacuum dominance

- Vacuum energy density

$$\frac{\rho_{\rm v}}{\rho} \sim \left(\frac{\ell_{\rm p}}{R}\right)^2 \times \exp\left(2t/R\right) \longrightarrow t_{\rm br} \sim \mathsf{R} \ln(\mathsf{R}/\ell_{\rm P}) \qquad \rho_{\rm v} \sim \rho$$

- In a few milliseconds the vacuum take control over the energy density of the compact object

$$t_{\rm br} \sim 10^{-3} s$$

Vacuum dominance

- Vacuum energy density

$$\frac{\rho_{\rm v}}{\rho} \sim \left(\frac{\ell_{\rm p}}{R}\right)^2 \times \exp\left(2t/R\right) \longrightarrow t_{\rm br} \sim \mathsf{R}\ln(\mathsf{R}/\ell_{\rm P}) \qquad \rho_{\rm v} \sim \rho$$

- In a few milliseconds the vacuum take control over the energy density of the compact object

$$t_{\rm br} \sim 10^{-3} s \qquad\qquad \rho_{\rm v} \sim 10^{14-15} {\rm g/cm^3}$$

Vacuum dominance

- Vacuum energy density

$$\frac{\rho_{\rm v}}{\rho} \sim \left(\frac{\ell_{\rm p}}{R}\right)^2 \times \exp\left(2t/R\right) \longrightarrow t_{\rm br} \sim \mathsf{R}\ln(\mathsf{R}/\ell_{\rm P}) \qquad \rho_{\rm v} \sim \rho$$

- In a few milliseconds the vacuum take control over the energy density of the compact object

$$t_{\rm br} \sim 10^{-3}s \qquad\qquad \rho_{\rm v} \sim 10^{14-15}{\rm g/cm}^3$$

- The unstable phase must be detained by backreaction effects which should be responsible to bring the vacuum back to some stationary regime.

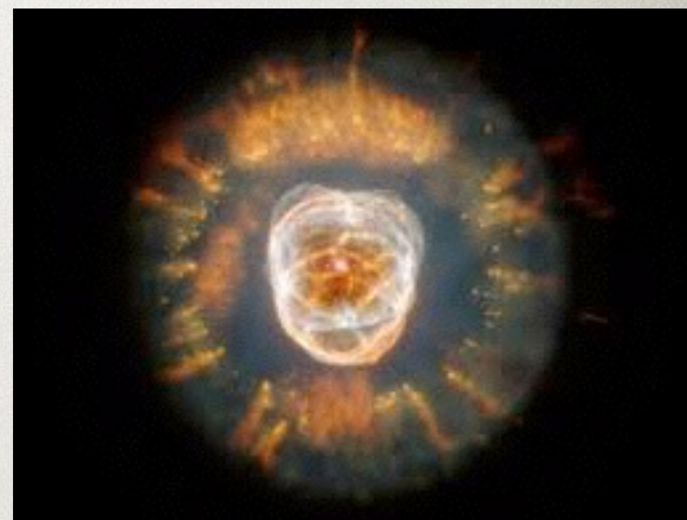
Awaking the vacuum

Awaking the vacuum

But what is the fate of the star?

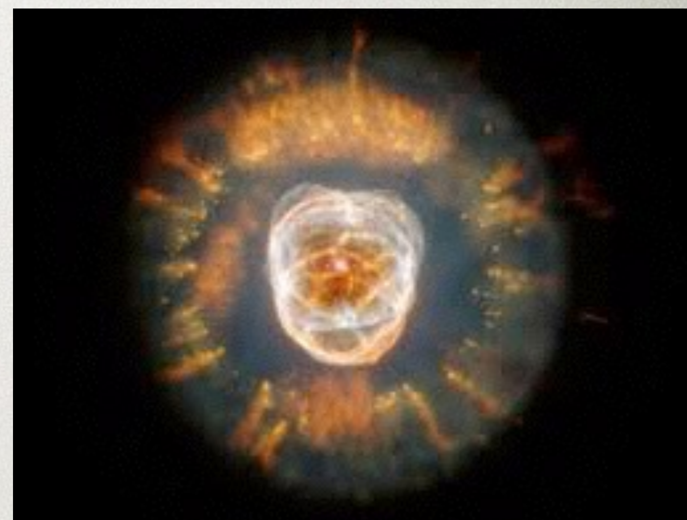
Awaking the vacuum

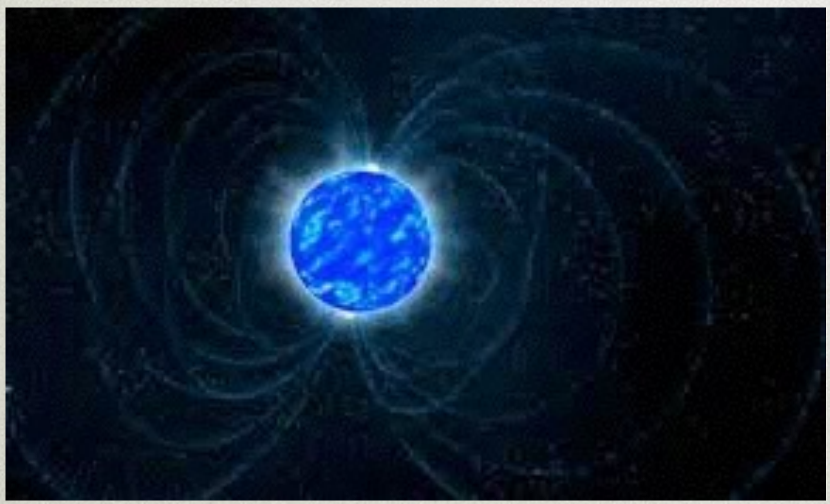
But what is the fate of the star?



Awaking the vacuum

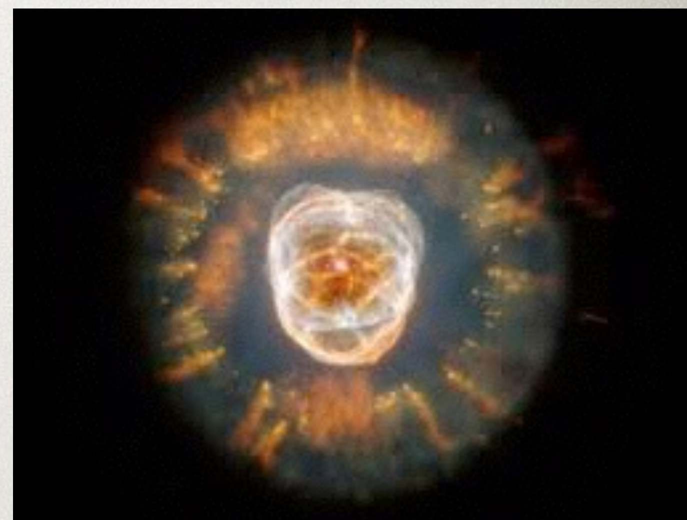
But what is the fate of the star?

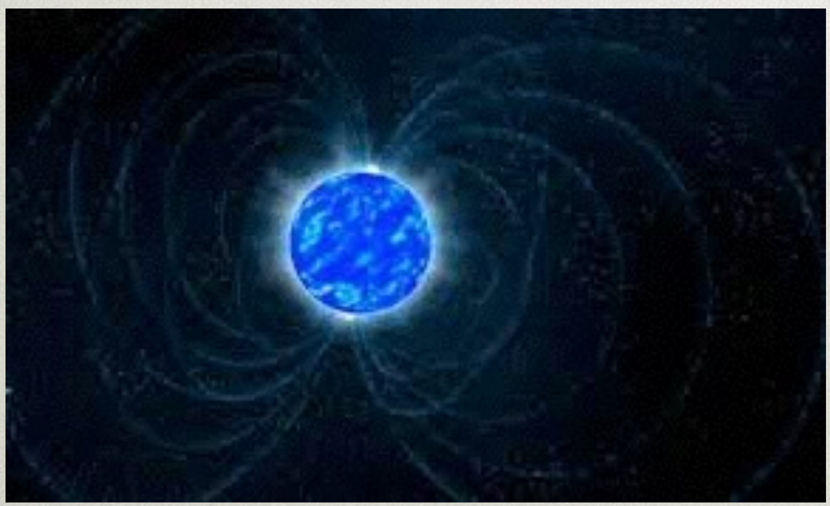


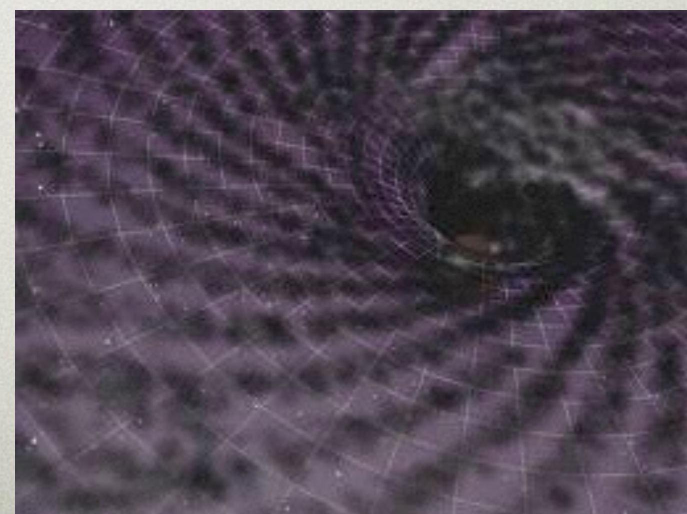


Awaking the vacuum

But what is the fate of the star?





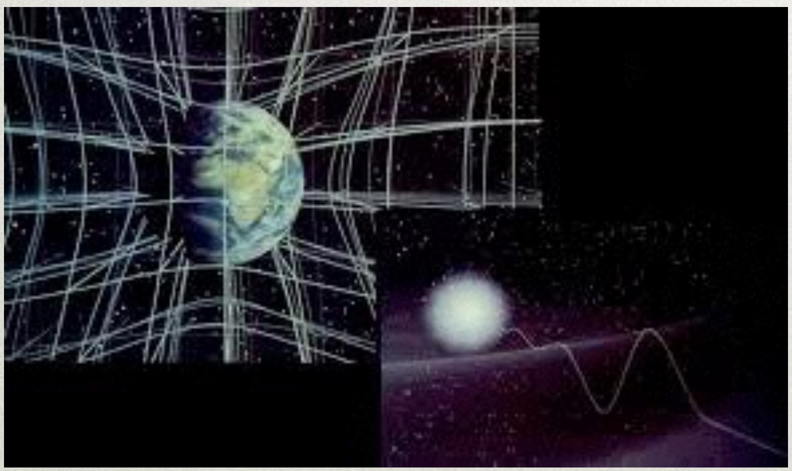


Awaking the vacuum

Awaking the vacuum

- Einstein semiclassical equation:

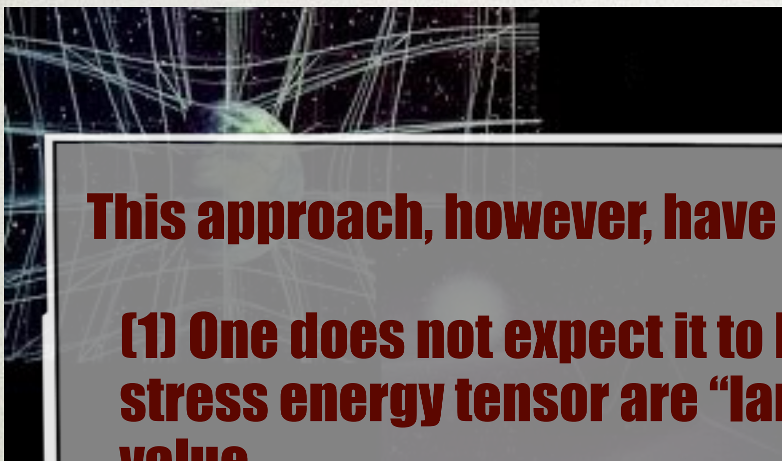
$$R_{ab} - \frac{1}{2}g_{ab}R = 8\pi\langle T_{ab}\rangle_\omega$$



Awaking the vacuum

- Einstein semiclassical equation:

$$R_{ab} - \frac{1}{2} g_{ab} R = 8\pi \langle T_{ab} \rangle_\omega$$



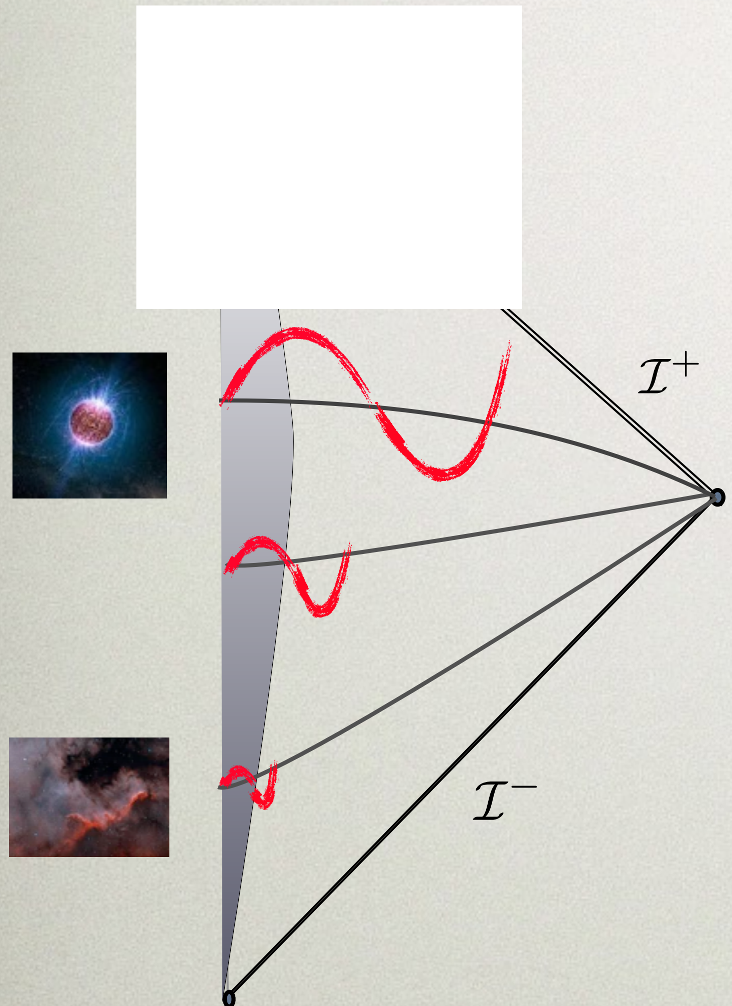
This approach, however, have many difficulties, e.g.,

(1) One does not expect it to be valid when the fluctuations of the stress energy tensor are "large" when compared to its mean value

(2) Even if/when it is valid, it is, in general, prohibitively difficult to solve it.

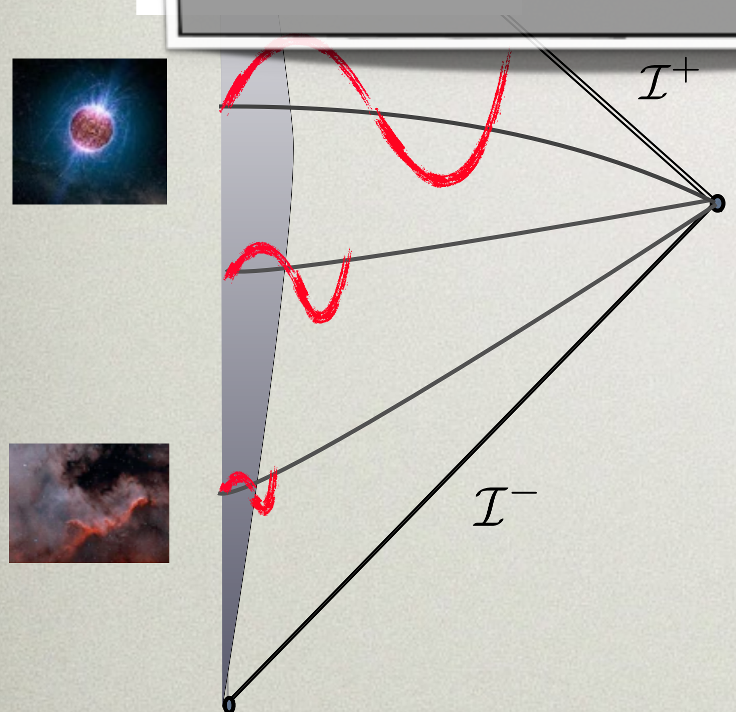
⋮

Awaking the vacuum



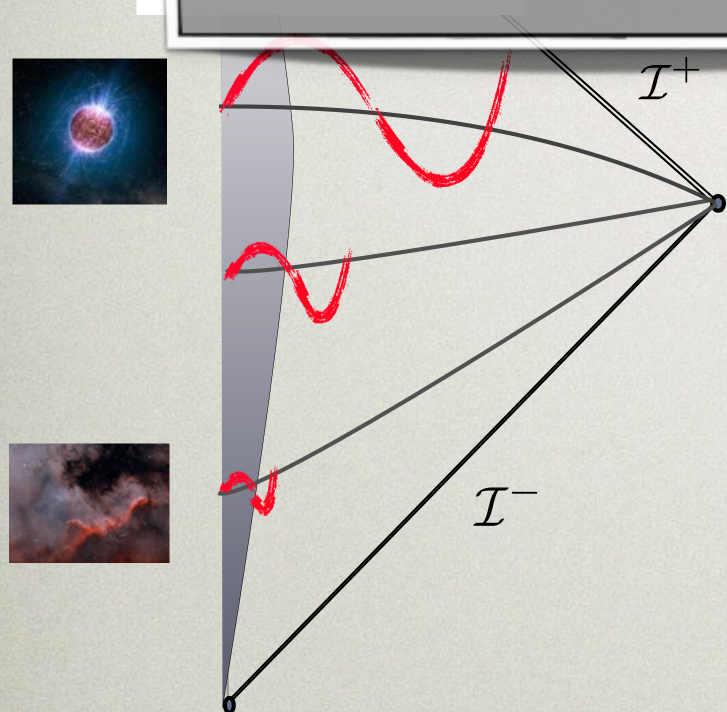
Awaking the vacuum

it seems reasonable to believe that quantum fluctuations amplified enough to menace the stability of relativistic stars cannot remain ``quantum'' for too long.



Awaking the vacuum

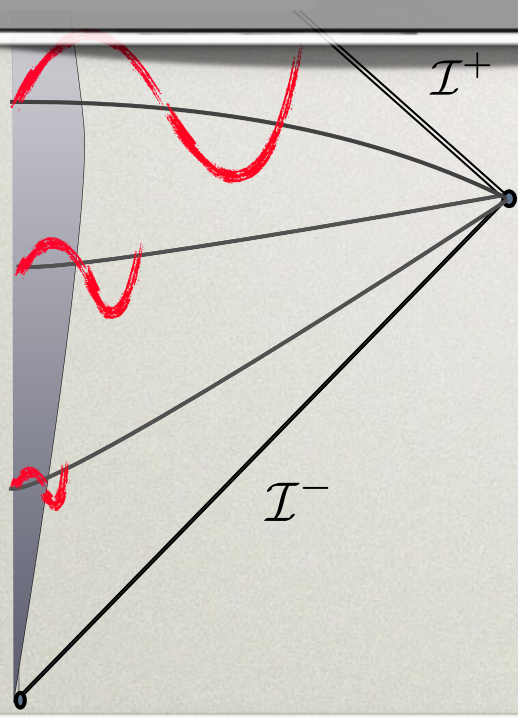
it seems reasonable to believe that quantum fluctuations amplified enough to menace the stability of relativistic stars cannot remain ``quantum" for too long.



If the quantum phase ends before vacuum fluctuations dominate the system, we expect backreaction to be well described by the CLASSICAL general-relativistic equations.

Awaking the vacuum

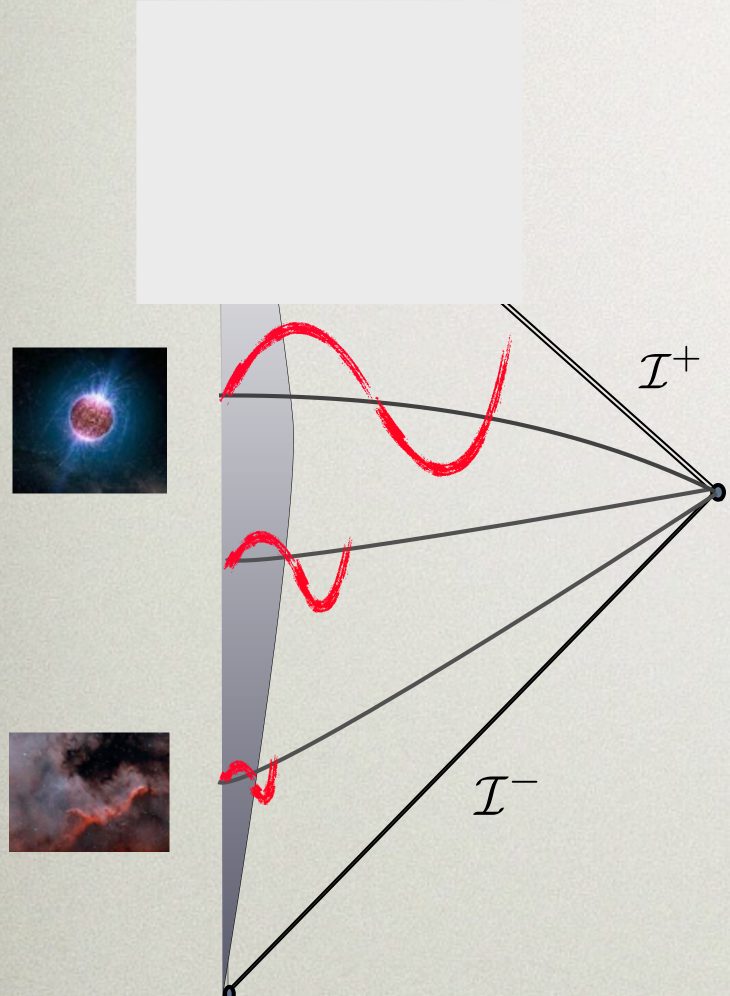
it seems reasonable to believe that quantum fluctuations amplified enough to menace the stability of relativistic stars cannot remain ``quantum'' for too long.



If the quantum phase ends before vacuum fluctuations dominate the system, we expect backreaction to be well described by the <u>CLASSICAL</u> general-relativistic equations.

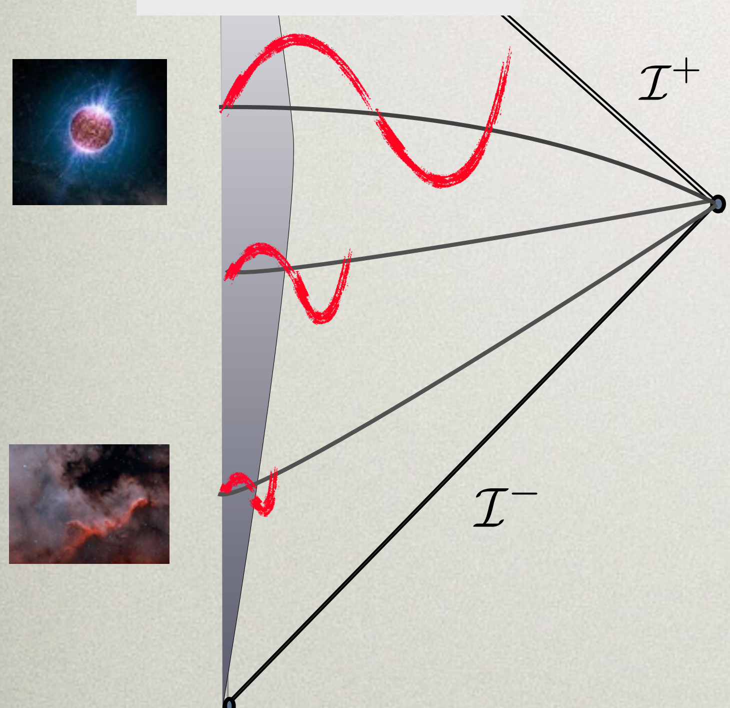
The relationship between the quantum and classical description of the instability was analyzed by R. Mendes, G. Matsas, and D. Vanzella PRD (1014)

Awaking the vacuum

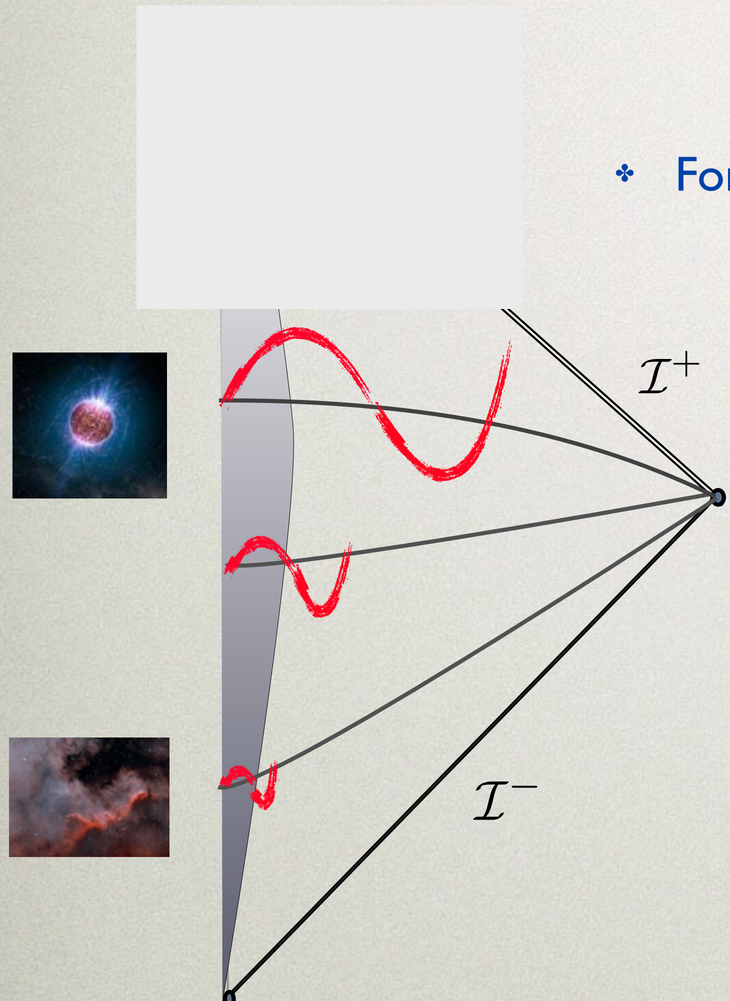


Awaking the vacuum

* For this quantum to classical transition to occur one must have:



Awaking the vacuum

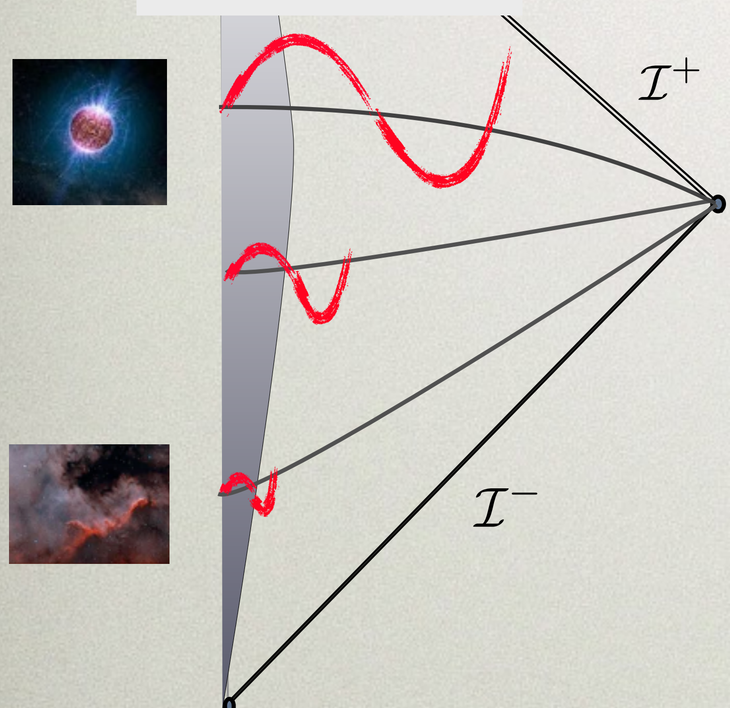


- For this quantum to classical transition to occur one must have:

  (1) The appearance of of certain “classical" correlations

Awaking the vacuum

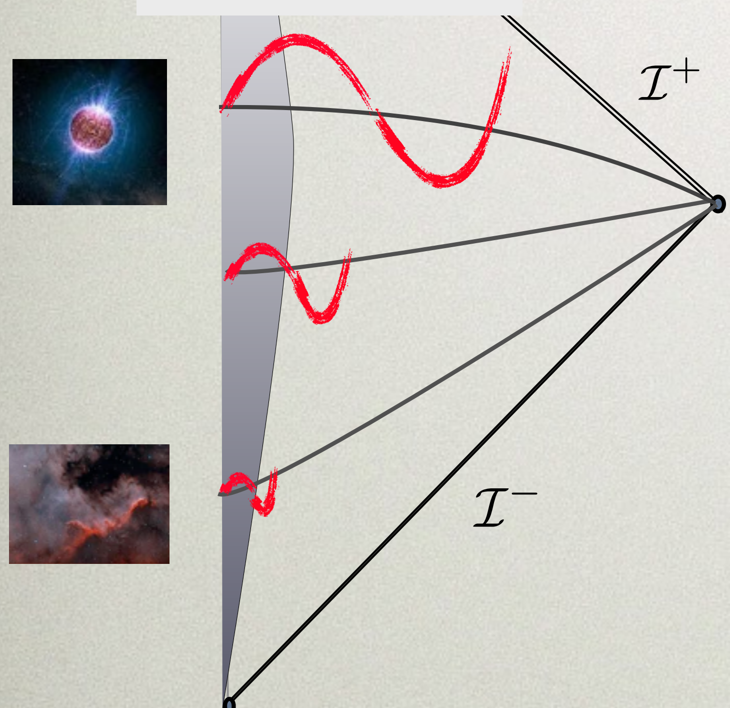
- For this quantum to classical transition to occur one must have:



(1) The appearance of of certain “classical" correlations

(2) Loss of quantum coherence

Awaking the vacuum



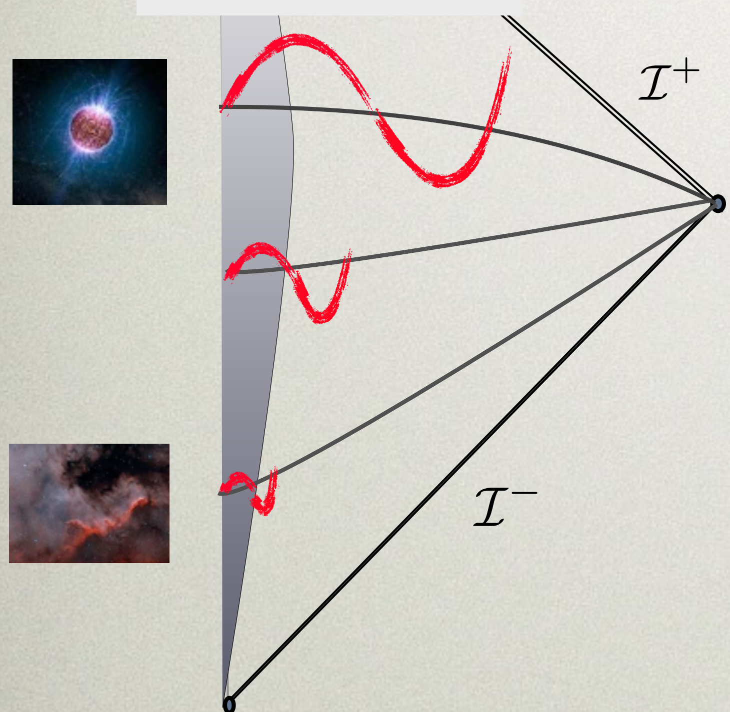
- For this quantum to classical transition to occur one must have:

(1) The appearance of of certain "classical" correlations

(2) Loss of quantum coherence

System
$\mathcal{S}$

Awaking the vacuum



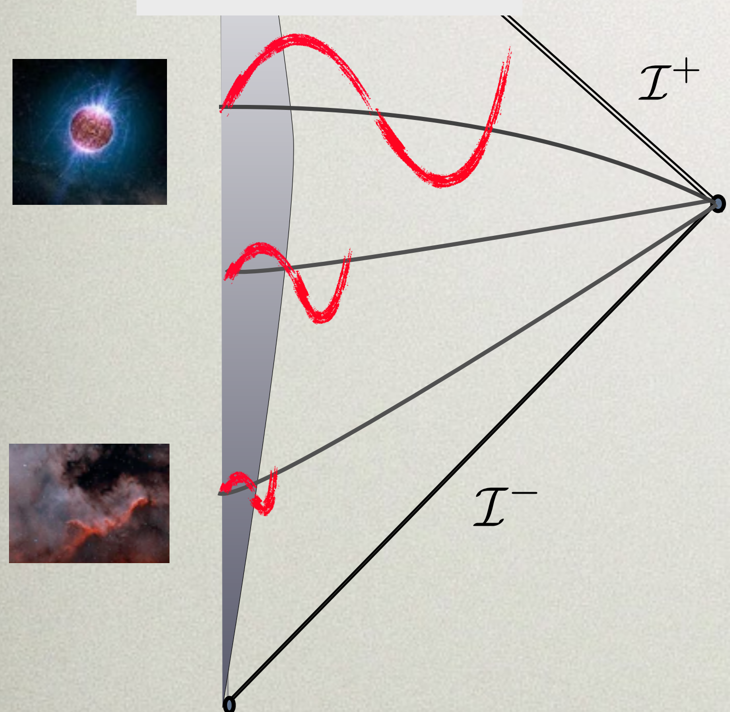
- For this quantum to classical transition to occur one must have:

  (1) The appearance of of certain “classical" correlations

  (2) Loss of quantum coherence

  System +
  $\mathcal{S}$

Awaking the vacuum



- For this quantum to classical transition to occur one must have:

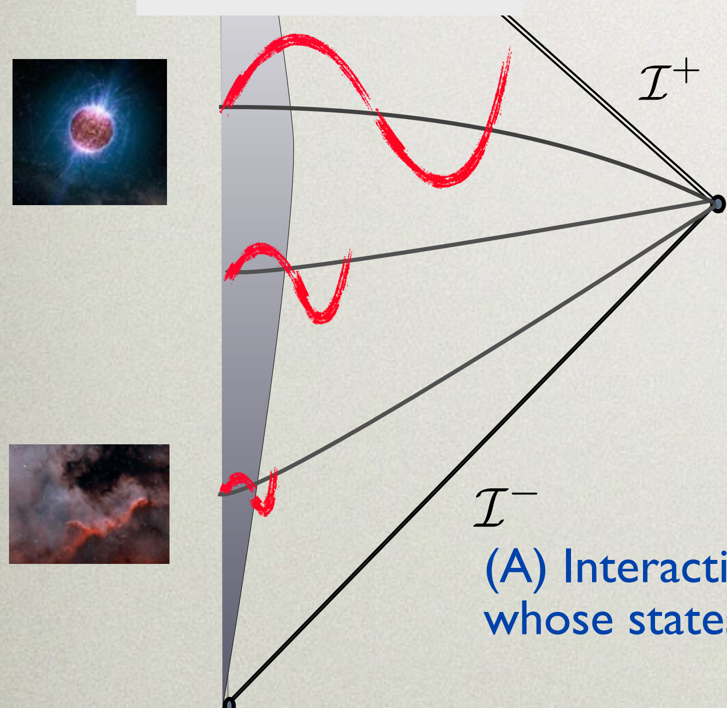
  (1) The appearance of of certain "classical" correlations

  (2) Loss of quantum coherence

  System + Environment

  $\mathcal{S}$ $\mathcal{E}$

Awaking the vacuum



- For this quantum to classical transition to occur one must have:

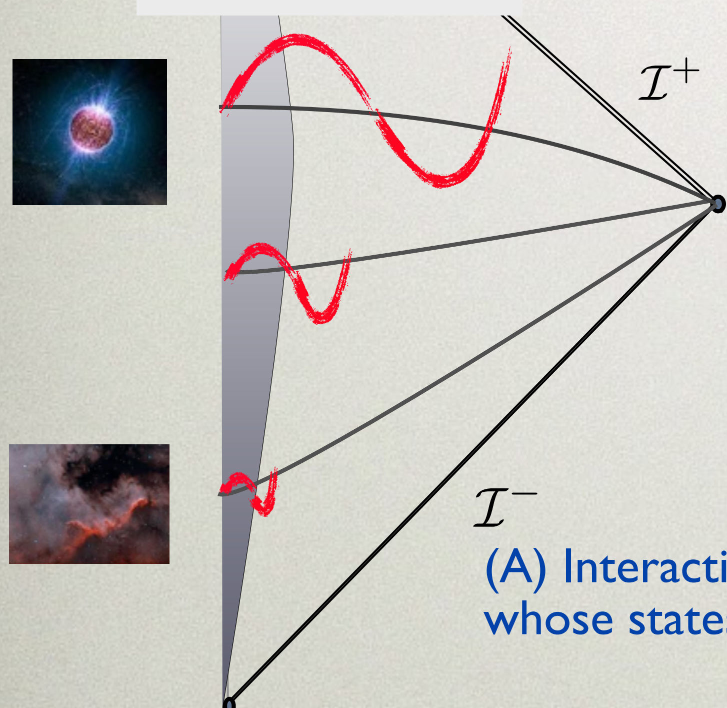
(1) The appearance of of certain "classical" correlations

(2) Loss of quantum coherence

System $\mathcal{S}$ + Environment $\mathcal{E}$

(A) Interaction between $\mathcal{S}$ and $\mathcal{E}$ selects a preferred basis of $\mathcal{S}$ whose states are "stable" (in spite of the interaction with $\mathcal{E}$)

Awaking the vacuum



- For this quantum to classical transition to occur one must have:

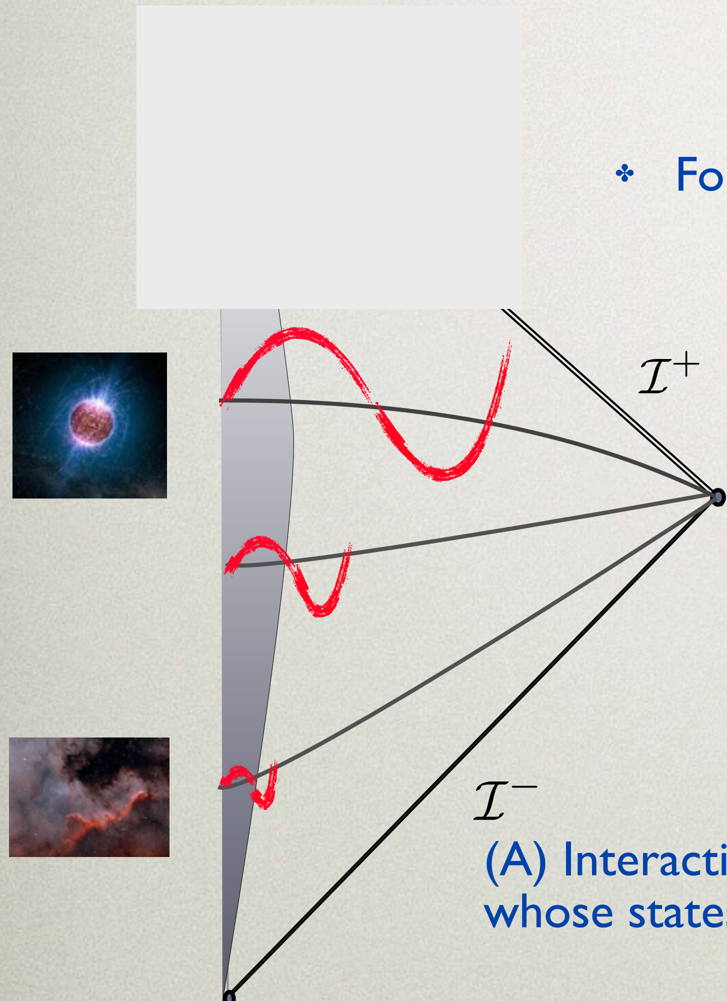
(1) The appearance of of certain “classical" correlations

(2) Loss of quantum coherence

System $\mathcal{S}$ + Environment $\mathcal{E}$

(A) Interaction between $\mathcal{S}$ and $\mathcal{E}$ selects a preferred basis of $\mathcal{S}$ whose states are “stable” (in spite of the interaction with $\mathcal{E}$)

Awaking the vacuum



- For this quantum to classical transition to occur one must have:

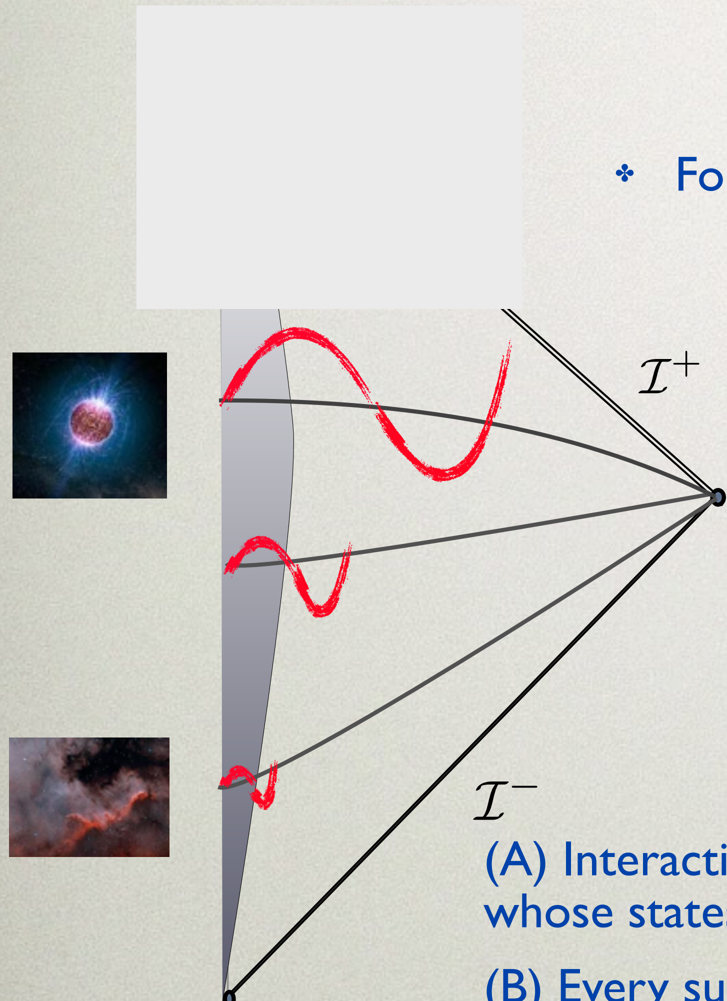
(1) The appearance of of certain “classical" correlations

(2) Loss of quantum coherence

System + Environment
$\mathcal{S}$ $\mathcal{E}$

(A) Interaction between $\mathcal{S}$ and $\mathcal{E}$ selects a preferred basis of $\mathcal{S}$ whose states are “stable” (in spite of the interaction with $\mathcal{E}$) $\longrightarrow$ Pointer States

Awaking the vacuum



- For this quantum to classical transition to occur one must have:

(1) The appearance of of certain “classical" correlations

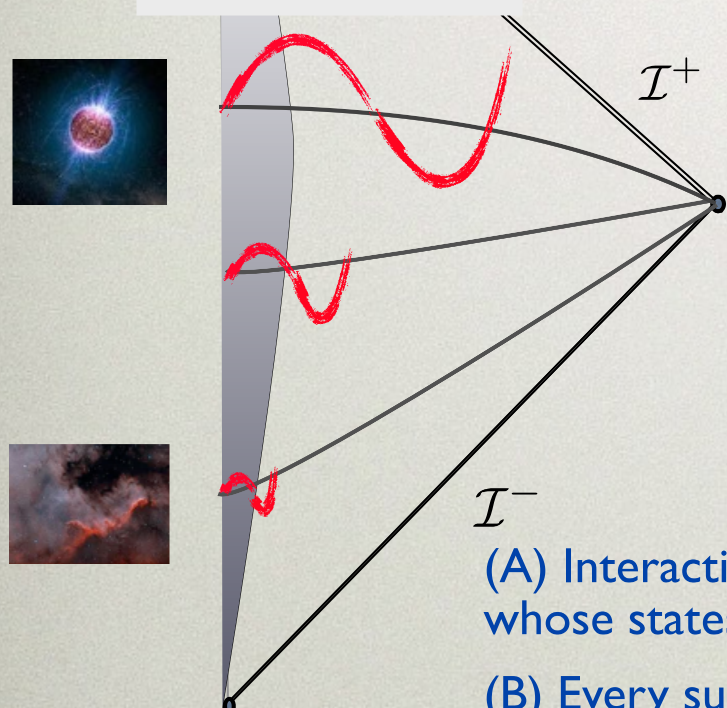
(2) Loss of quantum coherence

System + Environment
$\mathcal{S}$ $\mathcal{E}$

(A) Interaction between $\mathcal{S}$ and $\mathcal{E}$ selects a preferred basis of $\mathcal{S}$ whose states are “stable” (in spite of the interaction with $\mathcal{E}$) $\longrightarrow$ Pointer States

(B) Every superposition of pointer states evolves to a statistical mixture of the pointer states (i.e. every density matrix become diagonal in the pointer state basis)

Awaking the vacuum



- For this quantum to classical transition to occur one must have:

(1) The appearance of of certain "classical" correlations

(2) Loss of quantum coherence

System $\mathcal{S}$ + Environment $\mathcal{E}$

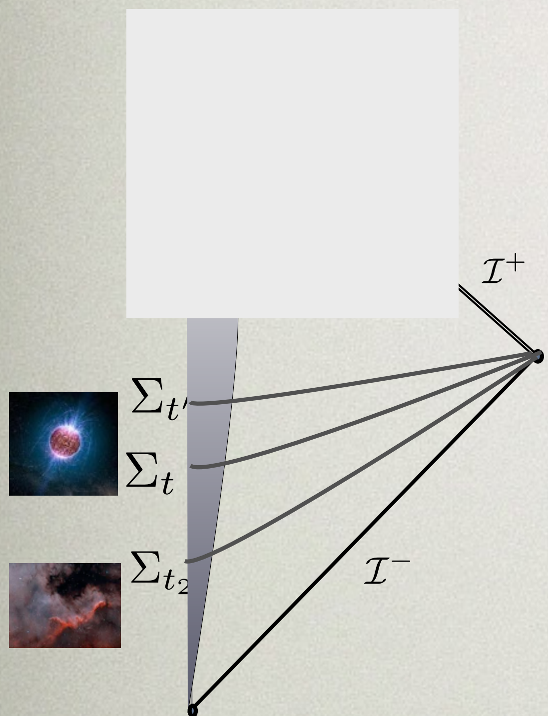
(A) Interaction between $\mathcal{S}$ and $\mathcal{E}$ selects a preferred basis of $\mathcal{S}$ whose states are "stable" (in spite of the interaction with $\mathcal{E}$) $\rightarrow$ Pointer States

(B) Every superposition of pointer states evolves to a statistical mixture of the pointer states (i.e. every density matrix become diagonal in the pointer state basis)

W. Zurek, RMP (2003), W. Zurek, Nature Phys. (2009)

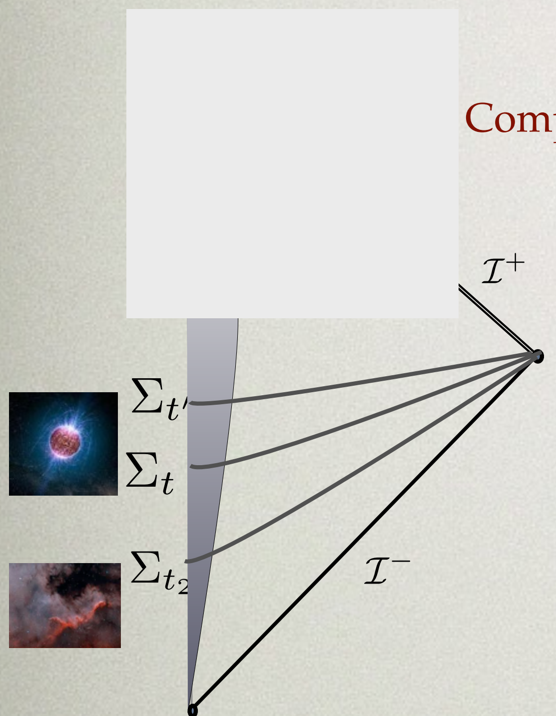
From quantum to classical

A.Landulfo, W. Lima, G. Mastsas, and D. Vanzella, PRD (2015)



From quantum to classical

A. Landulfo, W. Lima, G. Mastsas, and D. Vanzella, PRD (2015)

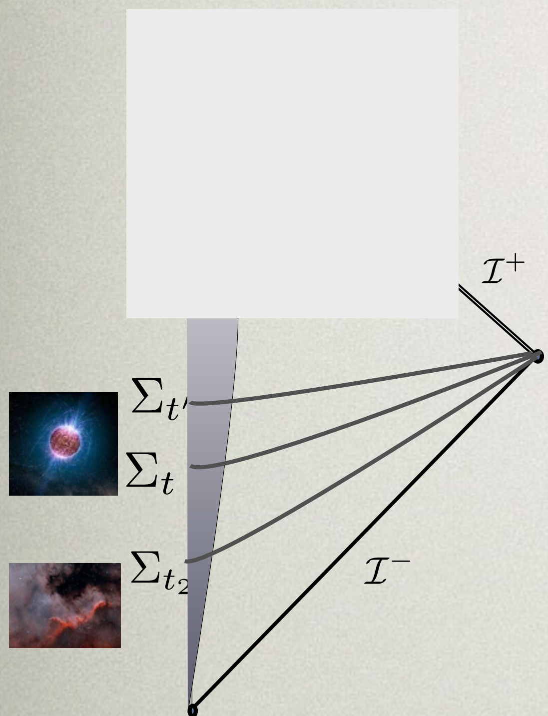


Complete orthonormal set of solutions of KG equation

$$\{v^{(+)}_{\varpi l\mu}, v^{(-)}_{\varpi l\mu}\},\ \{w^{(+)}_{\Omega}, w^{(-)}_{\Omega}\}$$

From quantum to classical

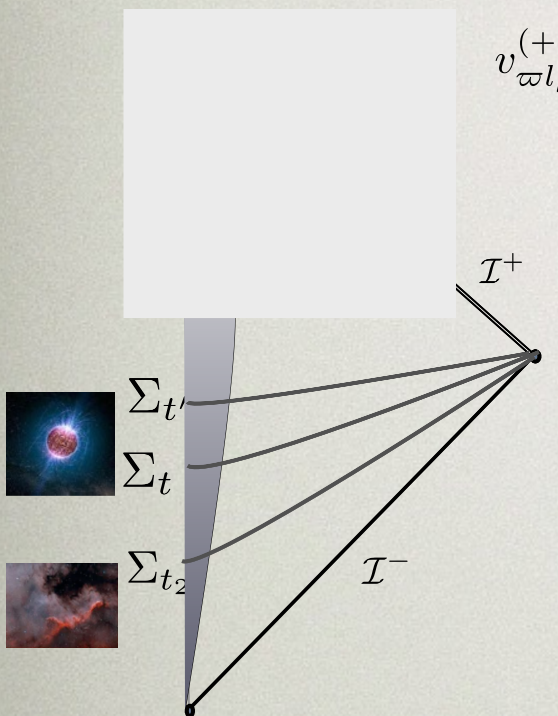
A. Landulfo, W. Lima, G. Mastsas, and D. Vanzella, PRD (2015)



From quantum to classical

A. Landulfo, W. Lima, G. Mastsas, and D. Vanzella, PRD (2015)

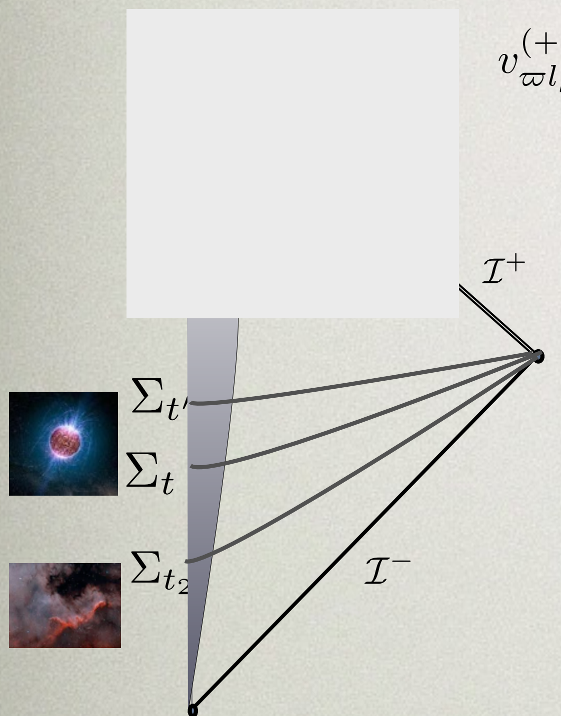
$$v^{(+)}_{\varpi l \mu} = e^{-i\varpi t}\,\frac{F_{\varpi l}(\chi)}{\sqrt{2\varpi}\,r(\chi)}Y_{l\mu}(\theta,\phi),$$



From quantum to classical

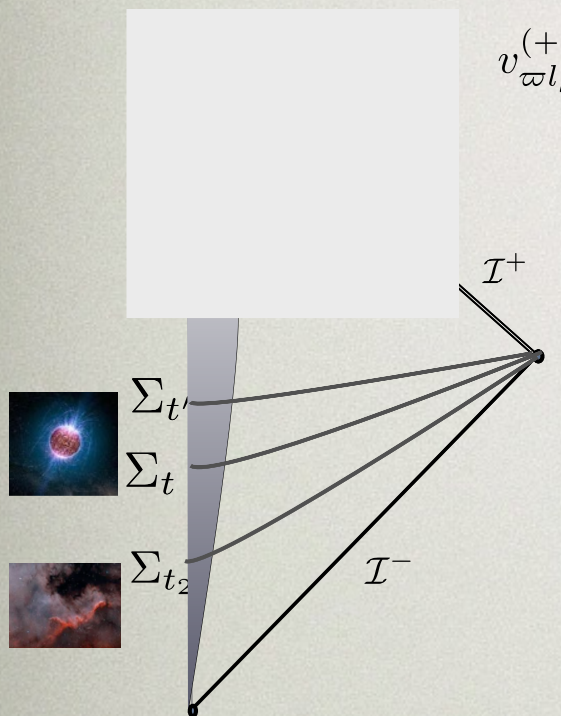
A. Landulfo, W. Lima, G. Mastsas, and D. Vanzella, PRD (2015)

$$v^{(+)}_{\varpi l \mu} = e^{-i\varpi t}\,\frac{F_{\varpi l}(\chi)}{\sqrt{2\varpi}\,r(\chi)} Y_{l\mu}(\theta,\phi), \qquad w^{(+)}_{\Omega} = \frac{e^{\Omega t - i\pi/4} + e^{-\Omega t + i\pi/4}}{\sqrt{4\Omega}}\,\frac{F_{\Omega}(\chi)}{r(\chi)} Y_{00}(\theta,\varphi)$$



From quantum to classical

A. Landulfo, W. Lima, G. Mastsas, and D. Vanzella, PRD (2015)

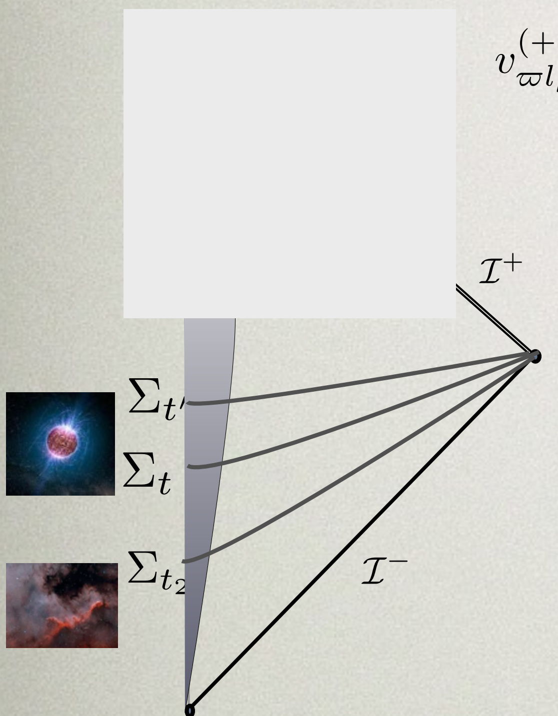


$$v^{(+)}_{\varpi l\mu} = e^{-i\varpi t}\frac{F_{\varpi l}(\chi)}{\sqrt{2\varpi}\, r(\chi)}Y_{l\mu}(\theta,\phi), \qquad w^{(+)}_{\Omega} = \frac{e^{\Omega t - i\pi/4} + e^{-\Omega t + i\pi/4}}{\sqrt{4\Omega}}\frac{F_\Omega(\chi)}{r(\chi)}Y_{00}(\theta,\varphi)$$

$$\hat{\Phi} = \sum_{l\mu}\int d\varpi[\hat{b}_{\varpi l\mu}v^{(+)}_{\varpi l\mu} + \hat{b}^\dagger_{\varpi l\mu}v^{(-)}_{\varpi l\mu}] + [\hat{a}_\Omega w^{(+)}_\Omega + \hat{a}^\dagger_\Omega w^{(-)}_\Omega]$$

From quantum to classical

A. Landulfo, W. Lima, G. Mastsas, and D. Vanzella, PRD (2015)



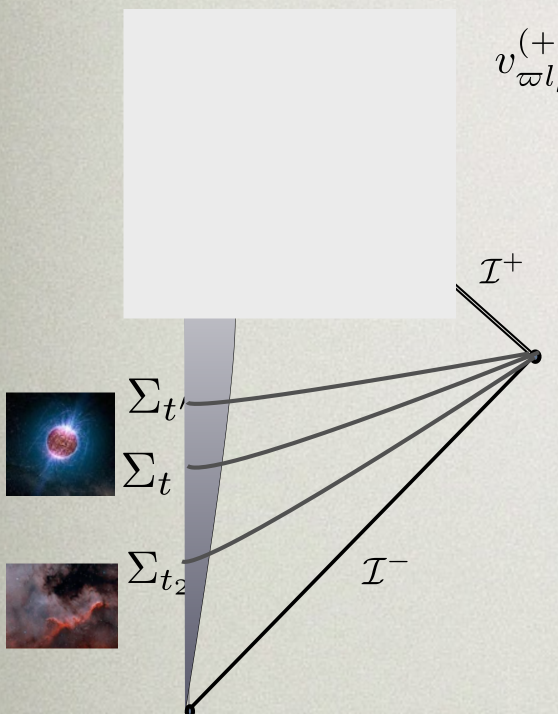
$$v^{(+)}_{\varpi l \mu} = e^{-i\varpi t}\,\frac{F_{\varpi l}(\chi)}{\sqrt{2\varpi}\,r(\chi)}Y_{l\mu}(\theta,\phi), \qquad w^{(+)}_{\Omega} = \frac{e^{\Omega t - i\pi/4} + e^{-\Omega t + i\pi/4}}{\sqrt{4\Omega}}\,\frac{F_{\Omega}(\chi)}{r(\chi)}Y_{00}(\theta,\varphi)$$

$$\hat{\Phi} = \sum_{l\mu}\int d\varpi[\hat{b}_{\varpi l\mu}v^{(+)}_{\varpi l\mu} + \hat{b}^{\dagger}_{\varpi l\mu}v^{(-)}_{\varpi l\mu}] + [\hat{a}_{\Omega}w^{(+)}_{\Omega} + \hat{a}^{\dagger}_{\Omega}w^{(-)}_{\Omega}]$$

Hamiltonian: $$\hat{H} = \hat{H}_s + \hat{H}_u$$

From quantum to classical

A. Landulfo, W. Lima, G. Mastsas, and D. Vanzella, PRD (2015)



$$v^{(+)}_{\varpi l \mu} = e^{-i\varpi t}\frac{F_{\varpi l}(\chi)}{\sqrt{2\varpi}\,r(\chi)}Y_{l\mu}(\theta,\phi), \qquad w^{(+)}_{\Omega} = \frac{e^{\Omega t - i\pi/4} + e^{-\Omega t + i\pi/4}}{\sqrt{4\Omega}}\frac{F_{\Omega}(\chi)}{r(\chi)}Y_{00}(\theta,\varphi)$$

$$\hat{\Phi} = \sum_{l\mu}\int d\varpi[\hat{b}_{\varpi l\mu}v^{(+)}_{\varpi l\mu} + \hat{b}^{\dagger}_{\varpi l\mu}v^{(-)}_{\varpi l\mu}] + [\hat{a}_{\Omega}w^{(+)}_{\Omega} + \hat{a}^{\dagger}_{\Omega}w^{(-)}_{\Omega}]$$

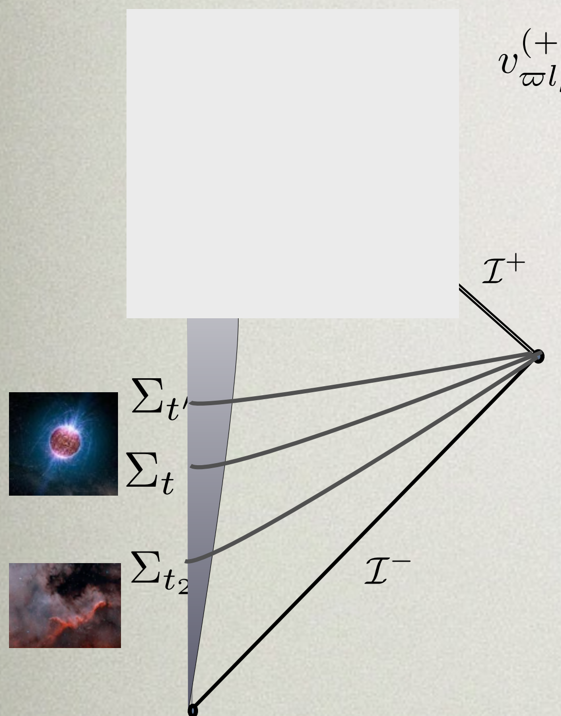
Hamiltonian:

$$\hat{H} = \hat{H}_s + \hat{H}_u$$

$$\hat{H}_s = \frac{1}{2}\sum_{l\mu}\int d\varpi[\hat{b}_{\varpi l\mu}\hat{b}^{\dagger}_{\varpi l\mu} + \hat{b}^{\dagger}_{\varpi l\mu^{\dagger}}\hat{b}_{\varpi l\mu}]\varpi$$

From quantum to classical

A. Landulfo, W. Lima, G. Mastsas, and D. Vanzella, PRD (2015)



$$v^{(+)}_{\varpi l\mu} = e^{-i\varpi t}\frac{F_{\varpi l}(\chi)}{\sqrt{2\varpi}\, r(\chi)} Y_{l\mu}(\theta,\phi), \qquad w^{(+)}_{\Omega} = \frac{e^{\Omega t - i\pi/4} + e^{-\Omega t + i\pi/4}}{\sqrt{4\Omega}} \frac{F_\Omega(\chi)}{r(\chi)} Y_{00}(\theta,\varphi)$$

$$\hat{\Phi} = \sum_{l\mu}\int d\varpi[\hat{b}_{\varpi l\mu} v^{(+)}_{\varpi l\mu} + \hat{b}^\dagger_{\varpi l\mu} v^{(-)}_{\varpi l\mu}] + [\hat{a}_\Omega w^{(+)}_\Omega + \hat{a}^\dagger_\Omega w^{(-)}_\Omega]$$

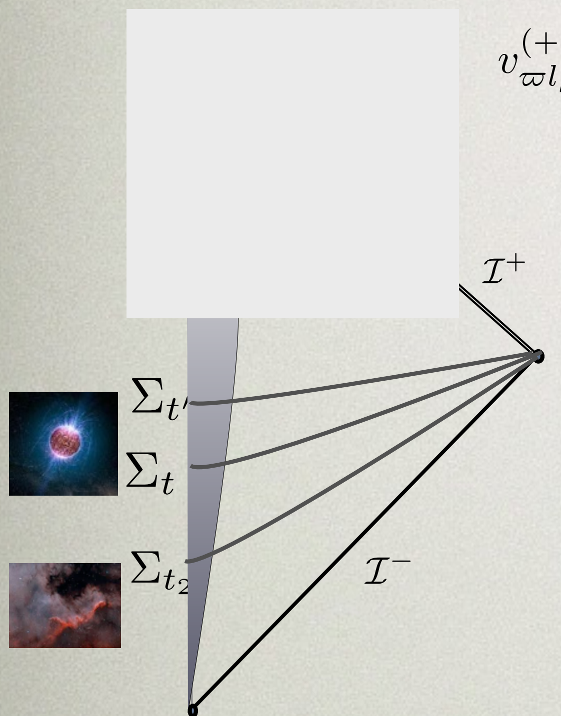
Hamiltonian: $$\hat{H} = \hat{H}_s + \hat{H}_u$$

$$\hat{H}_s = \frac{1}{2}\sum_{l\mu}\int d\varpi[\hat{b}_{\varpi l\mu}\hat{b}^\dagger_{\varpi l\mu} + \hat{b}^\dagger_{\varpi l\mu^\dagger}\hat{b}_{\varpi l\mu}]\varpi$$

$$\hat{H}_{\mathrm{u}} = -\frac{\Omega}{2}(\hat{a}_\Omega\hat{a}_\Omega + \hat{a}_\Omega{}^\dagger\hat{a}_\Omega{}^\dagger)$$

From quantum to classical

A. Landulfo, W. Lima, G. Mastsas, and D. Vanzella, PRD (2015)



$$v^{(+)}_{\varpi l\mu} = e^{-i\varpi t}\frac{F_{\varpi l}(\chi)}{\sqrt{2\varpi}\, r(\chi)} Y_{l\mu}(\theta,\phi), \qquad w^{(+)}_{\Omega} = \frac{e^{\Omega t - i\pi/4} + e^{-\Omega t + i\pi/4}}{\sqrt{4\Omega}}\frac{F_\Omega(\chi)}{r(\chi)} Y_{00}(\theta,\varphi)$$

$$\hat\Phi = \sum_{l\mu}\int d\varpi [\hat b_{\varpi l\mu} v^{(+)}_{\varpi l\mu} + \hat b^\dagger_{\varpi l\mu} v^{(-)}_{\varpi l\mu}] + [\hat a_\Omega w^{(+)}_\Omega + \hat a^\dagger_\Omega w^{(-)}_\Omega]$$

Hamiltonian: $$\hat H = \hat H_s + \hat H_u$$

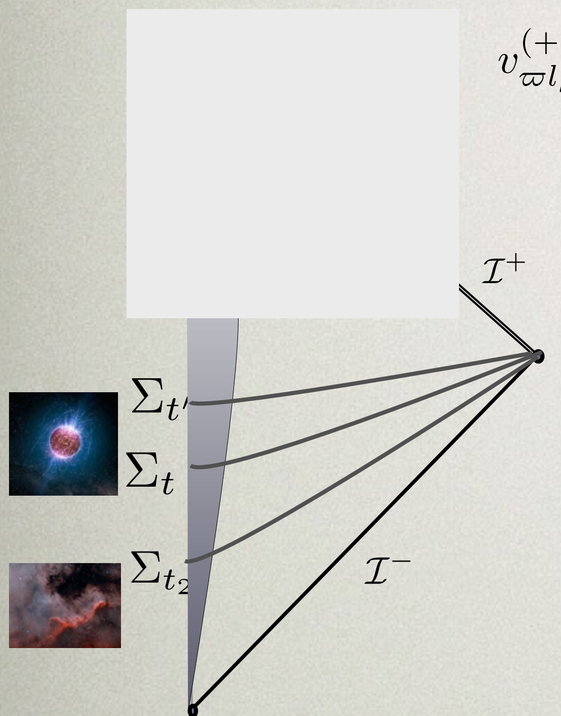
$$\hat H_s = \frac{1}{2}\sum_{l\mu}\int d\varpi [\hat b_{\varpi l\mu}\hat b^\dagger_{\varpi l\mu} + \hat b^\dagger_{\varpi l\mu^\dagger}\hat b_{\varpi l\mu}]\varpi$$

$$\hat H_{\rm u} = -\frac{\Omega}{2}(\hat a_\Omega \hat a_\Omega + \hat a_\Omega{}^\dagger \hat a_\Omega{}^\dagger)$$

$$\hat q_\Omega \equiv \frac{1}{\sqrt{2\Omega}}(\hat a_\Omega + \hat a_\Omega{}^\dagger)$$

From quantum to classical

A. Landulfo, W. Lima, G. Mastsas, and D. Vanzella, PRD (2015)



$$v^{(+)}_{\varpi l\mu} = e^{-i\varpi t}\frac{F_{\varpi l}(\chi)}{\sqrt{2\varpi}\, r(\chi)} Y_{l\mu}(\theta,\phi), \qquad w^{(+)}_{\Omega} = \frac{e^{\Omega t - i\pi/4} + e^{-\Omega t + i\pi/4}}{\sqrt{4\Omega}} \frac{F_\Omega(\chi)}{r(\chi)} Y_{00}(\theta,\varphi)$$

$$\hat\Phi = \sum_{l\mu}\int d\varpi[\hat b_{\varpi l\mu} v^{(+)}_{\varpi l\mu} + \hat b^\dagger_{\varpi l\mu} v^{(-)}_{\varpi l\mu}] + [\hat a_\Omega w^{(+)}_\Omega + \hat a^\dagger_\Omega w^{(-)}_\Omega]$$

Hamiltonian: $\hat H = \hat H_s + \hat H_u$

$$\hat H_s = \frac{1}{2}\sum_{l\mu}\int d\varpi[\hat b_{\varpi l\mu}\hat b^\dagger_{\varpi l\mu} + \hat b^\dagger_{\varpi l\mu^\dagger}\hat b_{\varpi l\mu}]\varpi$$

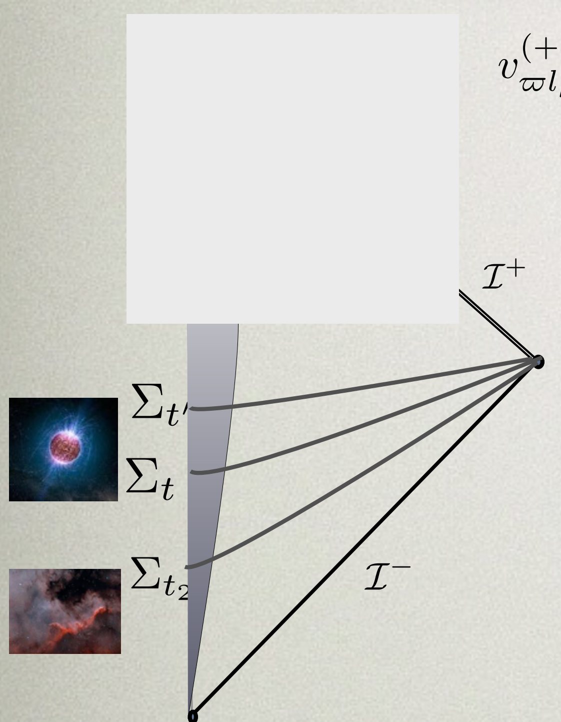
$$\hat H_{\mathrm{u}} = -\frac{\Omega}{2}(\hat a_\Omega \hat a_\Omega + \hat a_\Omega{}^\dagger \hat a_\Omega{}^\dagger)$$

$$\hat q_\Omega \equiv \frac{1}{\sqrt{2\Omega}}(\hat a_\Omega + \hat a_\Omega{}^\dagger) \qquad \hat p_\Omega \equiv -i\sqrt{\frac{\Omega}{2}}(\hat a_\Omega - \hat a_\Omega{}^\dagger)$$

From quantum to classical

A. Landulfo, W. Lima, G. Mastsas, and D. Vanzella, PRD (2015)

$$v^{(+)}_{\varpi l\mu} = e^{-i\varpi t}\frac{F_{\varpi l}(\chi)}{\sqrt{2\varpi}\, r(\chi)} Y_{l\mu}(\theta,\phi), \qquad w^{(+)}_{\Omega} = \frac{e^{\Omega t - i\pi/4} + e^{-\Omega t + i\pi/4}}{\sqrt{4\Omega}}\frac{F_{\Omega}(\chi)}{r(\chi)} Y_{00}(\theta,\varphi)$$



$$\hat{\Phi} = \sum_{l\mu}\int d\varpi [\hat{b}_{\varpi l\mu} v^{(+)}_{\varpi l\mu} + \hat{b}^{\dagger}_{\varpi l\mu} v^{(-)}_{\varpi l\mu}] + [\hat{a}_{\Omega} w^{(+)}_{\Omega} + \hat{a}^{\dagger}_{\Omega} w^{(-)}_{\Omega}]$$

Hamiltonian: $$\hat{H} = \hat{H}_s + \hat{H}_u$$

$$\hat{H}_s = \frac{1}{2}\sum_{l\mu}\int d\varpi [\hat{b}_{\varpi l\mu}\hat{b}^{\dagger}_{\varpi l\mu} + \hat{b}^{\dagger}_{\varpi l\mu^{\dagger}}\hat{b}_{\varpi l\mu}]\varpi$$

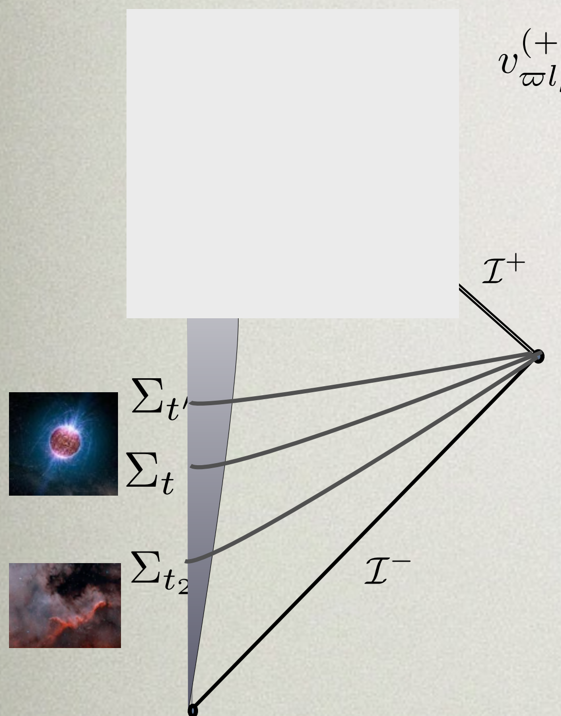
$$\hat{H}_{\mathrm{u}} = -\frac{\Omega}{2}(\hat{a}_{\Omega}\hat{a}_{\Omega} + \hat{a}_{\Omega}{}^{\dagger}\hat{a}_{\Omega}{}^{\dagger}) \rightarrow$$

$$\hat{q}_{\Omega} \equiv \frac{1}{\sqrt{2\Omega}}(\hat{a}_{\Omega} + \hat{a}_{\Omega}{}^{\dagger}) \qquad \hat{p}_{\Omega} \equiv -i\sqrt{\frac{\Omega}{2}}(\hat{a}_{\Omega} - \hat{a}_{\Omega}{}^{\dagger})$$

From quantum to classical

A. Landulfo, W. Lima, G. Mastsas, and D. Vanzella, PRD (2015)

$$v^{(+)}_{\varpi l\mu} = e^{-i\varpi t}\frac{F_{\varpi l}(\chi)}{\sqrt{2\varpi}\,r(\chi)}Y_{l\mu}(\theta,\phi), \qquad w^{(+)}_{\Omega} = \frac{e^{\Omega t - i\pi/4} + e^{-\Omega t + i\pi/4}}{\sqrt{4\Omega}}\frac{F_\Omega(\chi)}{r(\chi)}Y_{00}(\theta,\varphi)$$



$$\hat{\Phi} = \sum_{l\mu}\int d\varpi[\hat{b}_{\varpi l\mu}v^{(+)}_{\varpi l\mu} + \hat{b}^\dagger_{\varpi l\mu}v^{(-)}_{\varpi l\mu}] + [\hat{a}_\Omega w^{(+)}_\Omega + \hat{a}^\dagger_\Omega w^{(-)}_\Omega]$$

Hamiltonian: $$\hat{H} = \hat{H}_s + \hat{H}_u$$

$$\hat{H}_s = \frac{1}{2}\sum_{l\mu}\int d\varpi[\hat{b}_{\varpi l\mu}\hat{b}^\dagger_{\varpi l\mu} + \hat{b}^\dagger_{\varpi l\mu^\dagger}\hat{b}_{\varpi l\mu}]\varpi$$

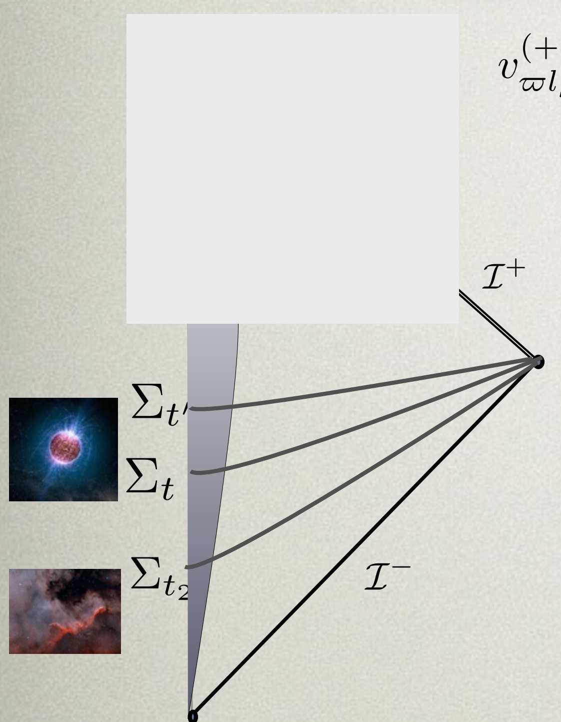
$$\hat{H}_{\mathrm{u}} = -\frac{\Omega}{2}(\hat{a}_\Omega\hat{a}_\Omega + \hat{a}_\Omega{}^\dagger\hat{a}_\Omega{}^\dagger) \rightarrow \hat{H}_{\mathrm{u}} = \frac{1}{2}\hat{p}^2_\Omega - \frac{\Omega^2}{2}\hat{q}^2_\Omega$$

$$\hat{q}_\Omega \equiv \frac{1}{\sqrt{2\Omega}}(\hat{a}_\Omega + \hat{a}_\Omega{}^\dagger) \qquad \hat{p}_\Omega \equiv -i\sqrt{\frac{\Omega}{2}}(\hat{a}_\Omega - \hat{a}_\Omega{}^\dagger)$$

From quantum to classical

A. Landulfo, W. Lima, G. Mastsas, and D. Vanzella, PRD (2015)

$$v^{(+)}_{\varpi l\mu} = e^{-i\varpi t}\frac{F_{\varpi l}(\chi)}{\sqrt{2\varpi}\, r(\chi)} Y_{l\mu}(\theta,\phi), \qquad w^{(+)}_{\Omega} = \frac{e^{\Omega t - i\pi/4} + e^{-\Omega t + i\pi/4}}{\sqrt{4\Omega}}\frac{F_\Omega(\chi)}{r(\chi)} Y_{00}(\theta,\varphi)$$



$$\hat\Phi = \sum_{l\mu}\int d\varpi[\hat b_{\varpi l\mu} v^{(+)}_{\varpi l\mu} + \hat b^\dagger_{\varpi l\mu} v^{(-)}_{\varpi l\mu}] + [\hat a_\Omega w^{(+)}_\Omega + \hat a^\dagger_\Omega w^{(-)}_\Omega]$$

Hamiltonian:

$$\hat H = \hat H_s + \hat H_u$$

$$\hat H_s = \frac{1}{2}\sum_{l\mu}\int d\varpi[\hat b_{\varpi l\mu}\hat b^\dagger_{\varpi l\mu} + \hat b^\dagger_{\varpi l\mu^\dagger}\hat b_{\varpi l\mu}]\varpi$$

$$\hat H_{\rm u} = -\frac{\Omega}{2}(\hat a_\Omega \hat a_\Omega + \hat a_\Omega{}^\dagger \hat a_\Omega{}^\dagger) \rightarrow \hat H_{\rm u} = \frac{1}{2}\hat p_\Omega^2 - \frac{\Omega^2}{2}\hat q_\Omega^2$$

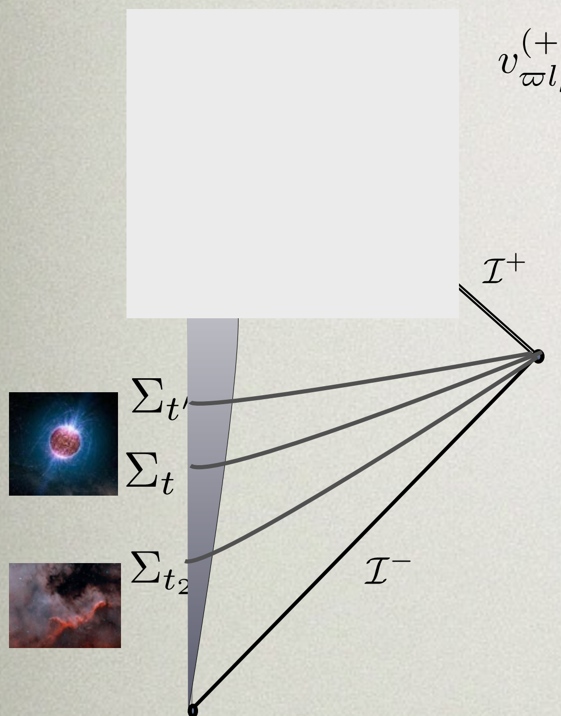
Upside down harmonic oscillator

$$\hat q_\Omega \equiv \frac{1}{\sqrt{2\Omega}}(\hat a_\Omega + \hat a_\Omega{}^\dagger) \qquad \hat p_\Omega \equiv -i\sqrt{\frac{\Omega}{2}}(\hat a_\Omega - \hat a_\Omega{}^\dagger)$$

FROM QUANTUM TO CLASSICAL

A.Landulfo, W. Lima, G. Mastsas, and D. Vanzella, PRD (2015)

$$v^{(+)}_{\varpi l\mu} = e^{-i\varpi t}\frac{F_{\varpi l}(\chi)}{\sqrt{2\varpi}\, r(\chi)} Y_{l\mu}(\theta,\phi), \qquad w^{(+)}_{\Omega} = \frac{e^{\Omega t - i\pi/4} + e^{-\Omega t + i\pi/4}}{\sqrt{4\Omega}}\frac{F_\Omega(\chi)}{r(\chi)} Y_{00}(\theta,\varphi)$$



Hamiltonian:

$$\hat{H} = \hat{H}_s + \hat{H}_u$$

$$\hat{H}_s = \frac{1}{2}\sum_{l\mu}\int d\varpi[\hat{b}_{\varpi l\mu}\hat{b}^\dagger_{\varpi l\mu} + \hat{b}^\dagger_{\varpi l\mu^\dagger}\hat{b}_{\varpi l\mu}]\varpi$$

$$\hat{H}_{\mathrm{u}} = -\frac{\Omega}{2}(\hat{a}_\Omega\hat{a}_\Omega + \hat{a}_\Omega{}^\dagger\hat{a}_\Omega{}^\dagger) \rightarrow \hat{H}_{\mathrm{u}} = \frac{1}{2}\hat{p}^2_\Omega - \frac{\Omega^2}{2}\hat{q}^2_\Omega$$

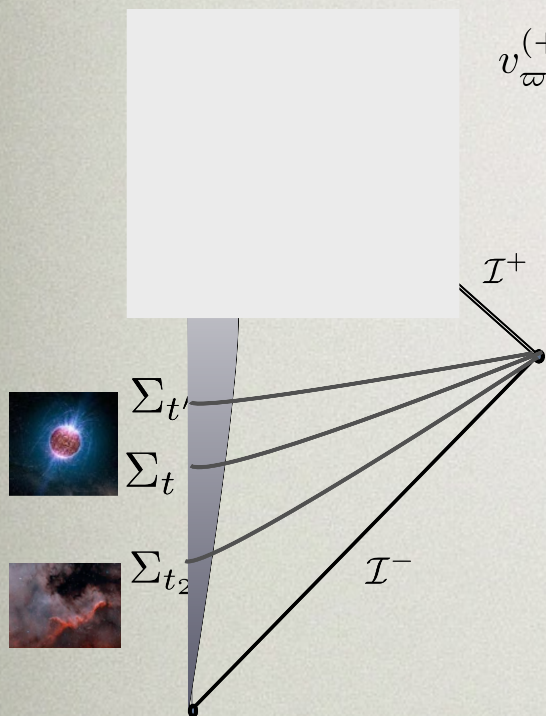
Upside down harmonic oscillator

$$\hat{q}_\Omega \equiv \frac{1}{\sqrt{2\Omega}}(\hat{a}_\Omega + \hat{a}_\Omega{}^\dagger) \qquad \hat{p}_\Omega \equiv -i\sqrt{\frac{\Omega}{2}}(\hat{a}_\Omega - \hat{a}_\Omega{}^\dagger)$$

From quantum to classical

A. Landulfo, W. Lima, G. Mastsas, and D. Vanzella, PRD (2015)

$$v^{(+)}_{\varpi l\mu} = e^{-i\varpi t}\frac{F_{\varpi l}(\chi)}{\sqrt{2\varpi}\,r(\chi)}Y_{l\mu}(\theta,\phi), \qquad w^{(+)}_{\Omega} = \frac{e^{\Omega t - i\pi/4} + e^{-\Omega t + i\pi/4}}{\sqrt{4\Omega}}\frac{F_\Omega(\chi)}{r(\chi)}Y_{00}(\theta,\varphi)$$



$$\hat{\Phi} = \sum_{l\mu}\int d\varpi[\hat{b}_{\varpi l\mu}v^{(+)}_{l\mu} + \hat{b}^\dagger_{\varpi l\mu}v^{(-)}_{l\mu}] + \left[\cosh(\Omega t)\hat{q}_\Omega + \frac{\sinh(\Omega t)}{\Omega}\hat{p}_\Omega\right]\frac{F_\Omega(\chi)}{r(\chi)}Y_{00}(\theta,\varphi)$$

Hamiltonian:

$$\hat{H} = \hat{H}_s + \hat{H}_u$$

$$\hat{H}_s = \frac{1}{2}\sum_{l\mu}\int d\varpi[\hat{b}_{\varpi l\mu}\hat{b}^\dagger_{\varpi l\mu} + \hat{b}^\dagger_{\varpi l\mu^\dagger}\hat{b}_{\varpi l\mu}]\varpi$$

$$\hat{H}_\mathrm{u} = -\frac{\Omega}{2}(\hat{a}_\Omega\hat{a}_\Omega + \hat{a}_\Omega{}^\dagger\hat{a}_\Omega{}^\dagger) \rightarrow \hat{H}_\mathrm{u} = \frac{1}{2}\hat{p}^2_\Omega - \frac{\Omega^2}{2}\hat{q}^2_\Omega$$

Upside down harmonic oscillator

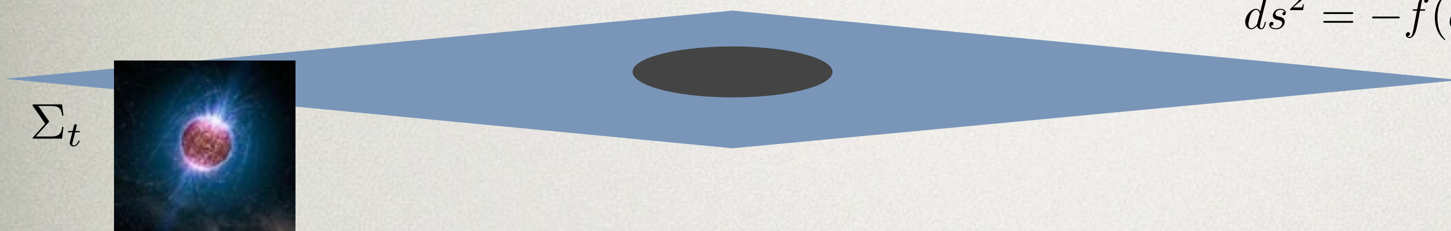
$$\hat{q}_\Omega \equiv \frac{1}{\sqrt{2\Omega}}(\hat{a}_\Omega + \hat{a}_\Omega{}^\dagger) \qquad \hat{p}_\Omega \equiv -i\sqrt{\frac{\Omega}{2}}(\hat{a}_\Omega - \hat{a}_\Omega{}^\dagger)$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

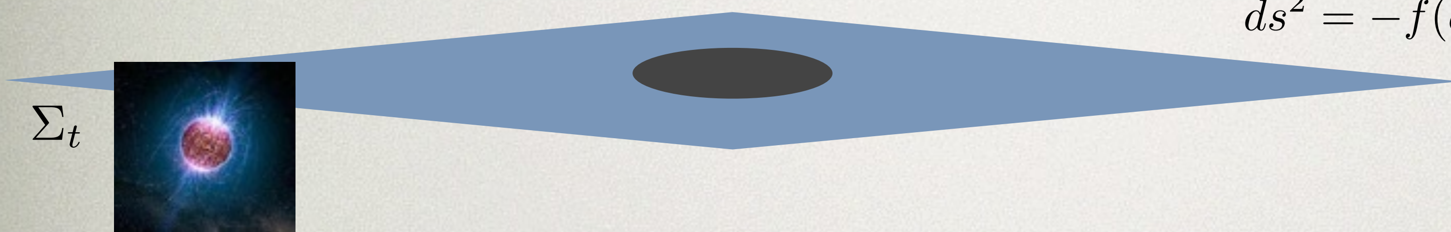
Σ_t



From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



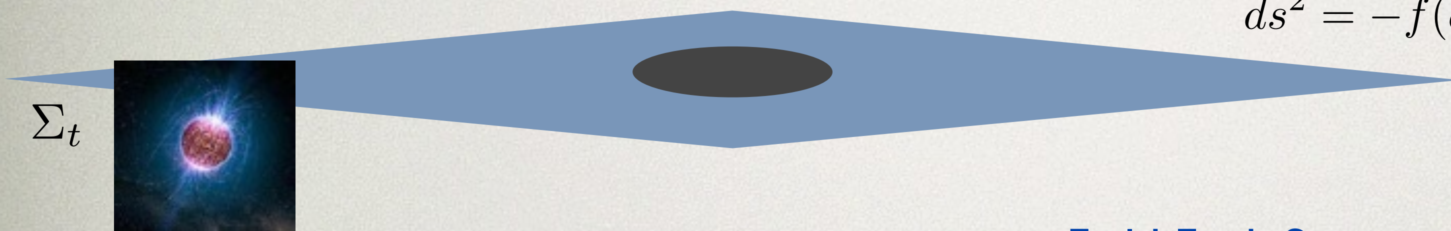
Σ_t

In this fixed background, the modes decouple and evolve independently.

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



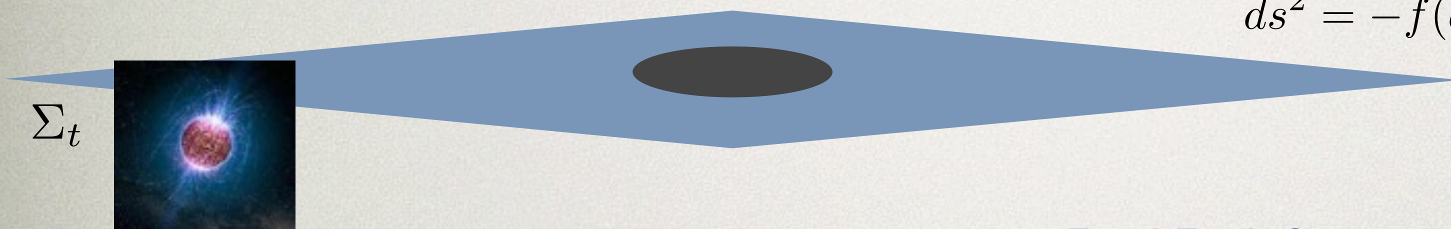
- Field Fock Space

In this fixed background, the modes decouple and evolve independently.

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Σ_t

In this fixed background, the modes decouple and evolve independently.

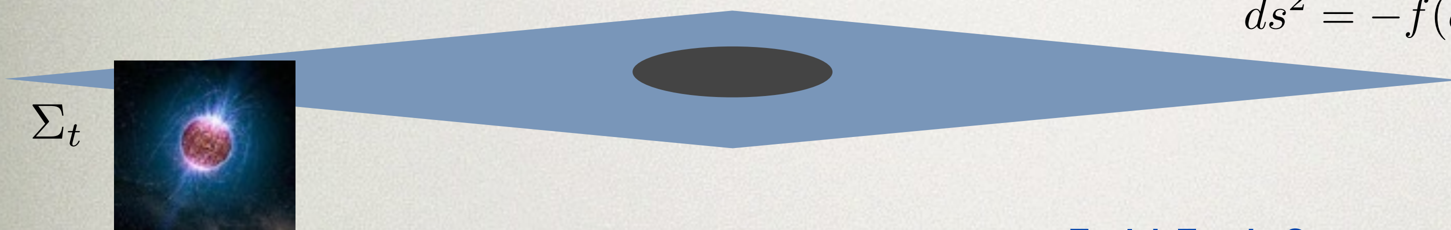
- Field Fock Space

$$\mathfrak{F}_s(\mathcal{H}_s) \otimes \mathfrak{F}_s(\mathcal{H}_u)$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Σ_t

In this fixed background, the modes decouple and evolve independently.

- Field Fock Space

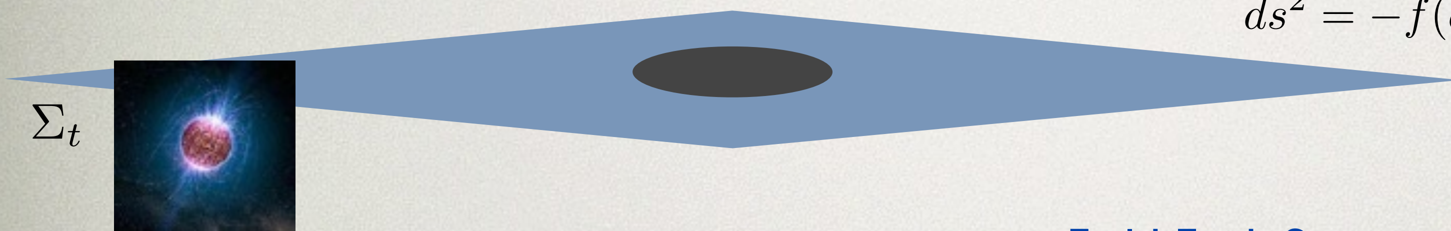
$$\mathfrak{F}_s(\mathcal{H}_s) \otimes \mathfrak{F}_s(\mathcal{H}_u)$$

- Unstable Phase Vacuum

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



In this fixed background, the modes decouple and evolve independently.

- Field Fock Space

$$\mathfrak{F}_s(\mathcal{H}_s) \otimes \mathfrak{F}_s(\mathcal{H}_u)$$

- Unstable Phase Vacuum

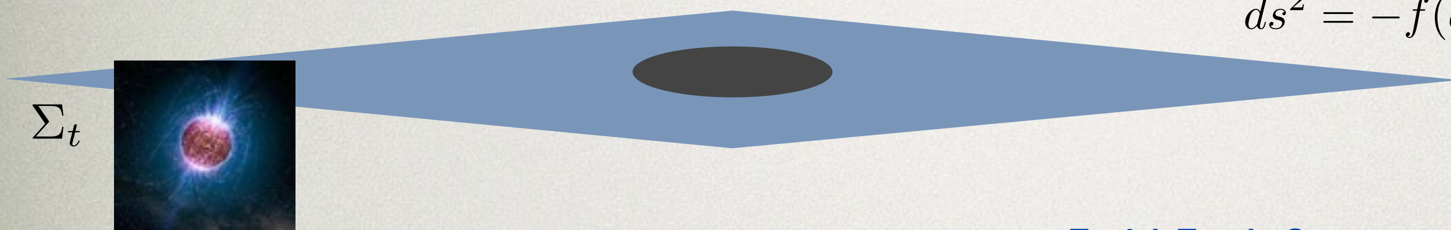
$$|0\rangle = |0_{\rm s}\rangle \otimes |0_{\rm u}\rangle$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t



In this fixed background, the modes decouple and evolve independently.

- Field Fock Space

$$\mathfrak{F}_s(\mathcal{H}_s) \otimes \mathfrak{F}_s(\mathcal{H}_u)$$

- Unitary evolution

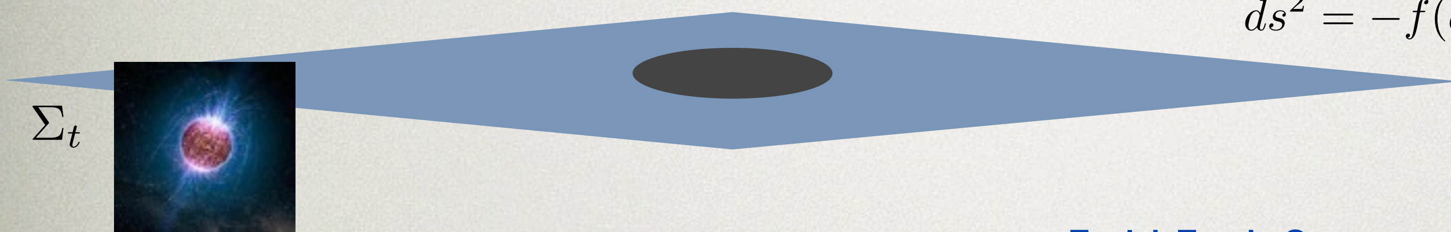
- Unstable Phase Vacuum

$$|0\rangle = |0_{\rm s}\rangle \otimes |0_{\rm u}\rangle$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



In this fixed background, the modes decouple and evolve independently.

- Field Fock Space

$$\mathfrak{F}_s(\mathcal{H}_s) \otimes \mathfrak{F}_s(\mathcal{H}_u)$$

- Unstable Phase Vacuum

$$|0\rangle = |0_\mathrm{s}\rangle \otimes |0_\mathrm{u}\rangle$$

- Unitary evolution

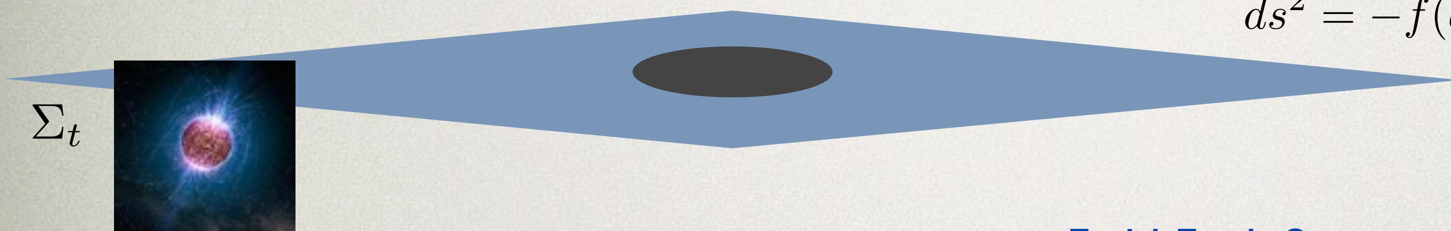
$$\hat{U}(t) \equiv e^{-it\hat{H}} = e^{-it\hat{H}_\mathrm{s}} \otimes e^{-it\hat{H}_\mathrm{u}}$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t



In this fixed background, the modes decouple and evolve independently.

- Field Fock Space

$$\mathfrak{F}_s(\mathcal{H}_s) \otimes \mathfrak{F}_s(\mathcal{H}_u)$$

- Unstable Phase Vacuum

$$|0\rangle = |0_{\rm s}\rangle \otimes |0_{\rm u}\rangle$$

- Unitary evolution

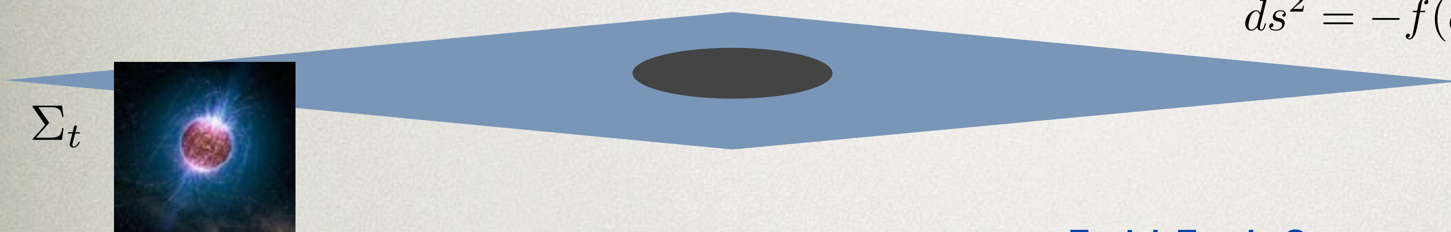
$$\hat{U}(t) \equiv e^{-it\hat{H}} = e^{-it\hat{H}_{\rm s}} \otimes e^{-it\hat{H}_{\rm u}}$$

$$|\eta(t)\rangle \equiv e^{-it\hat{H}_{\rm u}}|0_{\rm u}\rangle$$ Squeezed Vacuum

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



In this fixed background, the modes decouple and evolve independently.

- Field Fock Space

$$\mathfrak{F}_s(\mathcal{H}_s) \otimes \mathfrak{F}_s(\mathcal{H}_u)$$

- Unstable Phase Vacuum

$$|0\rangle = |0_\mathrm{s}\rangle \otimes |0_\mathrm{u}\rangle$$

- Unitary evolution

$$\hat{U}(t) \equiv e^{-it\hat{H}} = e^{-it\hat{H}_\mathrm{s}} \otimes e^{-it\hat{H}_\mathrm{u}}$$

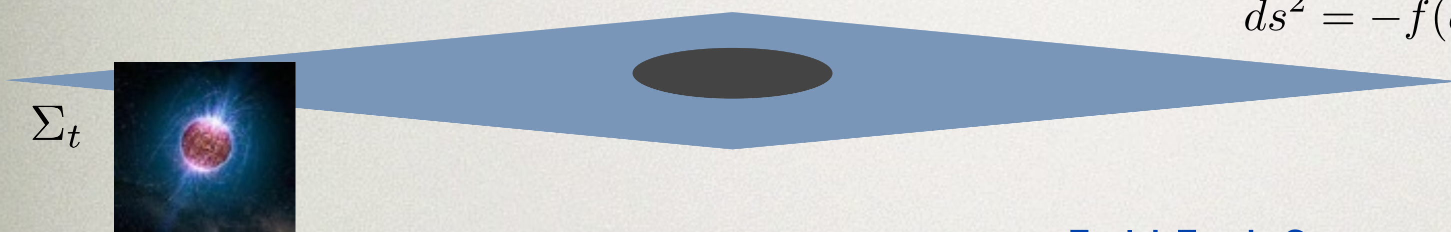
$|\eta(t)\rangle \equiv e^{-it\hat{H}_\mathrm{u}}|0_\mathrm{u}\rangle$ Squeezed Vacuum

$U_\mathrm{u}(t) \equiv e^{-it\hat{H}_\mathrm{u}}$ Squeezing Operator

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



In this fixed background, the modes decouple and evolve independently.

- Field Fock Space

$$\mathfrak{F}_s(\mathcal{H}_s) \otimes \mathfrak{F}_s(\mathcal{H}_u)$$

- Unstable Phase Vacuum

$$|0\rangle = |0_\mathrm{s}\rangle \otimes |0_\mathrm{u}\rangle$$

- Unitary evolution

$$\hat{U}(t) \equiv e^{-it\hat{H}} = e^{-it\hat{H}_\mathrm{s}} \otimes e^{-it\hat{H}_\mathrm{u}}$$

$$|\eta(t)\rangle \equiv e^{-it\hat{H}_\mathrm{u}}|0_\mathrm{u}\rangle$$ Squeezed Vacuum

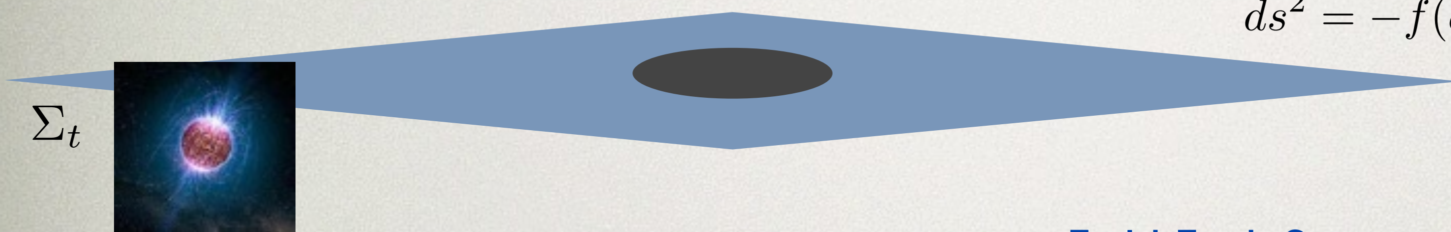
$$U_\mathrm{u}(t) \equiv e^{-it\hat{H}_\mathrm{u}}$$ Squeezing Operator

$$\eta(t,q) \equiv \langle q|\eta(t)\rangle = \left(\frac{a(t)}{\pi}\right)^{1/4} e^{i\alpha(t)q^2/2}.$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



In this fixed background, the modes decouple and evolve independently.

- Field Fock Space

$$\mathfrak{F}_s(\mathcal{H}_s) \otimes \mathfrak{F}_s(\mathcal{H}_u)$$

- Unstable Phase Vacuum

$$|0\rangle = |0_{\mathrm{s}}\rangle \otimes |0_{\mathrm{u}}\rangle$$

- Unitary evolution

$$\hat{U}(t) \equiv e^{-it\hat{H}} = e^{-it\hat{H}_{\mathrm{s}}} \otimes e^{-it\hat{H}_{\mathrm{u}}}$$

$$|\eta(t)\rangle \equiv e^{-it\hat{H}_{\mathrm{u}}}|0_{\mathrm{u}}\rangle$$ Squeezed Vacuum

$$U_{\mathrm{u}}(t) \equiv e^{-it\hat{H}_{\mathrm{u}}}$$ Squeezing Operator

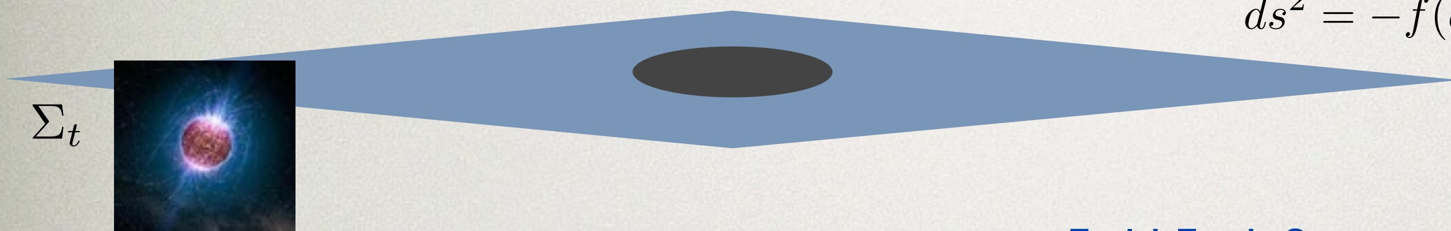
$$\eta(t,q) \equiv \langle q|\eta(t)\rangle = \left(\frac{a(t)}{\pi}\right)^{1/4} e^{i\alpha(t)q^2/2}.$$

$$\alpha(t) = i[a(t) + ib(t)]$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



In this fixed background, the modes decouple and evolve independently.

- Field Fock Space

$$\mathfrak{F}_s(\mathcal{H}_s) \otimes \mathfrak{F}_s(\mathcal{H}_u)$$

- Unstable Phase Vacuum

$$|0\rangle = |0_\mathrm{s}\rangle \otimes |0_\mathrm{u}\rangle$$

- Unitary evolution

$$\hat{U}(t) \equiv e^{-it\hat{H}} = e^{-it\hat{H}_\mathrm{s}} \otimes e^{-it\hat{H}_\mathrm{u}}$$

$$|\eta(t)\rangle \equiv e^{-it\hat{H}_\mathrm{u}}|0_\mathrm{u}\rangle$$ Squeezed Vacuum

$$U_\mathrm{u}(t) \equiv e^{-it\hat{H}_\mathrm{u}}$$ Squeezing Operator

$$\eta(t,q) \equiv \langle q|\eta(t)\rangle = \left(\frac{a(t)}{\pi}\right)^{1/4} e^{i\alpha(t)q^2/2}.$$

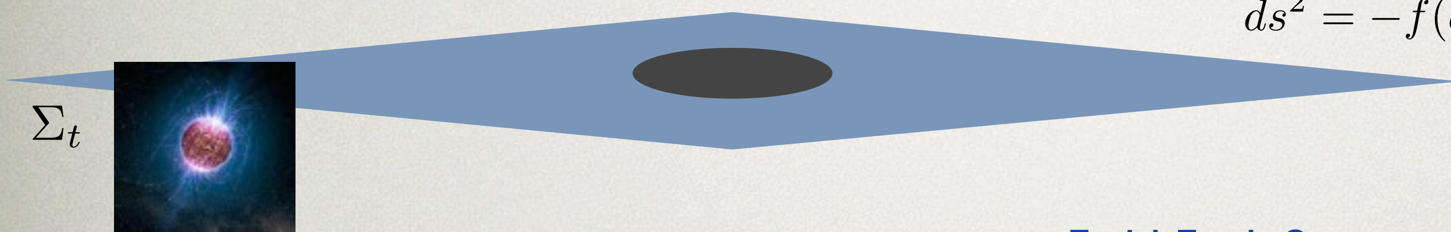
$$a(t) \equiv \frac{\Omega}{\cosh 2\Omega t}$$

$$\alpha(t) = i[a(t) + ib(t)]$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



In this fixed background, the modes decouple and evolve independently.

- Field Fock Space

$$\mathfrak{F}_s(\mathcal{H}_s) \otimes \mathfrak{F}_s(\mathcal{H}_u)$$

- Unstable Phase Vacuum

$$|0\rangle = |0_{\rm s}\rangle \otimes |0_{\rm u}\rangle$$

- Unitary evolution

$$\hat{U}(t) \equiv e^{-it\hat{H}} = e^{-it\hat{H}_{\rm s}} \otimes e^{-it\hat{H}_{\rm u}}$$

$$|\eta(t)\rangle \equiv e^{-it\hat{H}_{\rm u}}|0_{\rm u}\rangle$$ Squeezed Vacuum

$$U_{\rm u}(t) \equiv e^{-it\hat{H}_{\rm u}}$$ Squeezing Operator

$$\eta(t,q) \equiv \langle q|\eta(t)\rangle = \left(\frac{a(t)}{\pi}\right)^{1/4} e^{i\alpha(t)q^2/2}.$$

$$a(t) \equiv \frac{\Omega}{\cosh 2\Omega t}$$

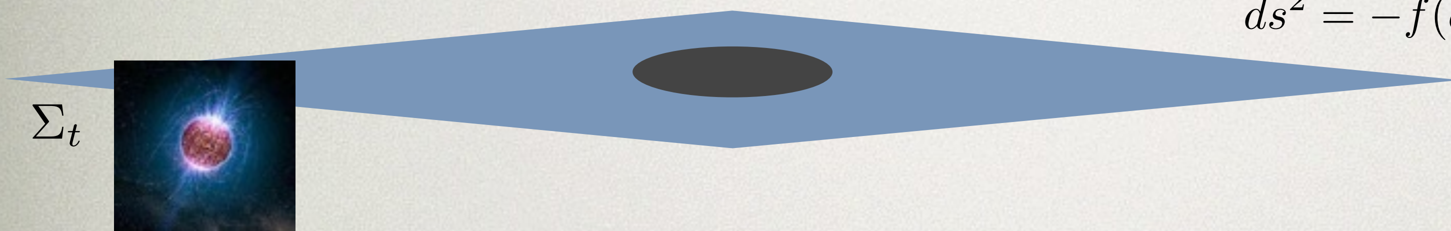
$$\alpha(t) = i[a(t) + ib(t)]$$

$$b(t) \equiv -\Omega \tanh 2\Omega t.$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



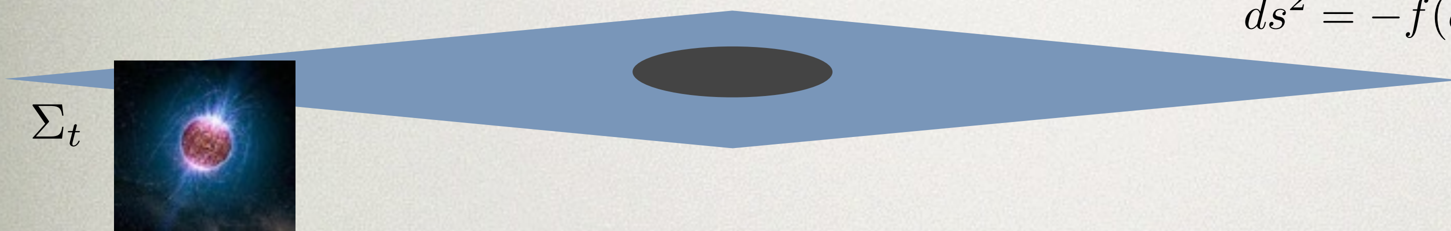
Σ_t

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t



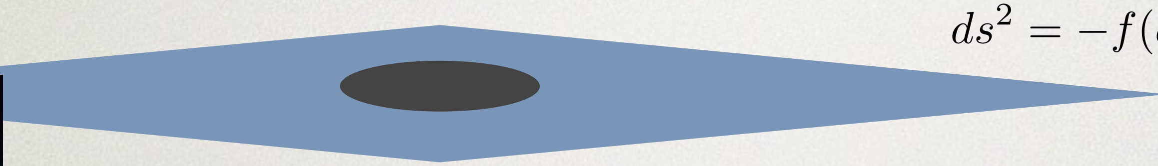
- Wigner Function

$$W(t,q,p) \equiv \frac{1}{2\pi}\int_{-\infty}^{+\infty} dy \varrho(t, q - y/2, q + y/2) e^{ipy},$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Σ_t

- Wigner Function

$$W(t,q,p) \equiv \frac{1}{2\pi}\int_{-\infty}^{+\infty} dy \varrho(t, q-y/2, q+y/2) e^{ipy},$$

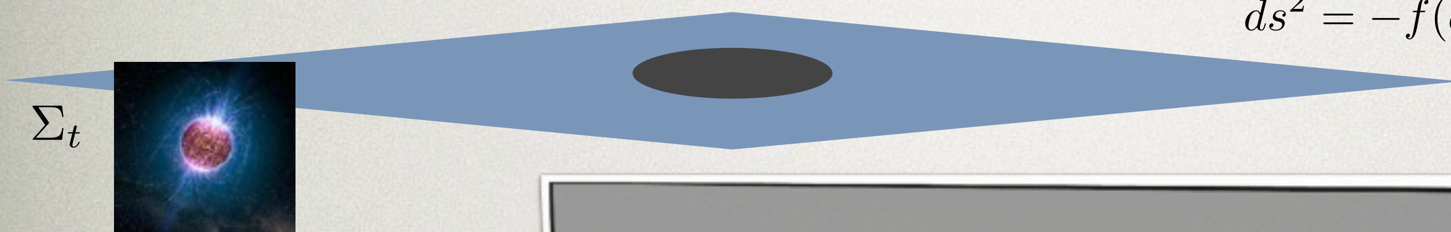
It is a very useful tool to analyze the classicalization of a quantum system

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t



It is a very useful tool to analyze the classicalization of a quantum system

- Wigner Function

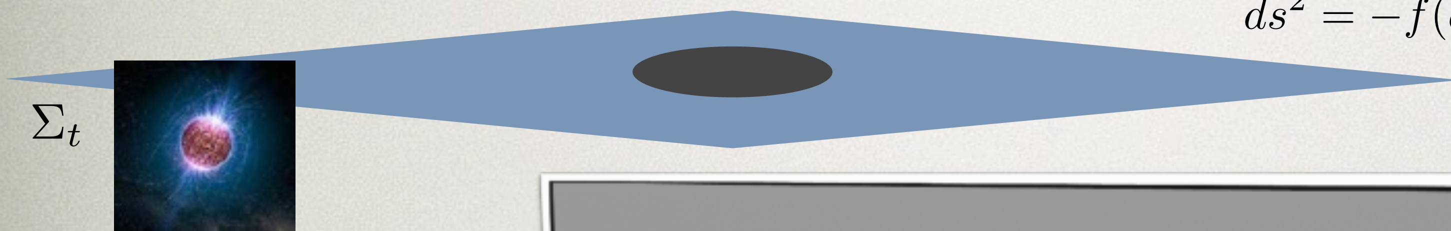
$$W(t,q,p) \equiv \frac{1}{2\pi}\int_{-\infty}^{+\infty} dy \varrho(t, q-y/2, q+y/2)e^{ipy},$$

- For pure states $|\psi\rangle$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



It is a very useful tool to analyze the classicalization of a quantum system

- Wigner Function

$$W(t,q,p) \equiv \frac{1}{2\pi}\int_{-\infty}^{+\infty} dy\varrho(t, q-y/2, q+y/2)e^{ipy},$$

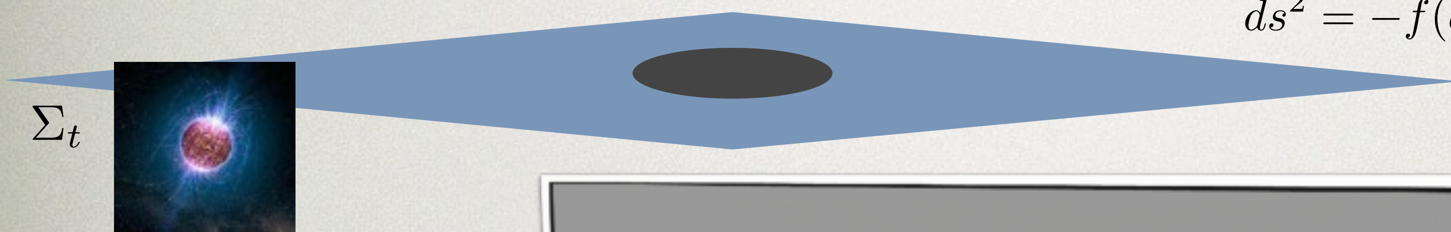
- For pure states $|\psi\rangle$

$$\int_{-\infty}^{+\infty} dqW(q,p) = |\tilde{\psi}(p)|^2,$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



It is a very useful tool to analyze the classicalization of a quantum system

- Wigner Function

$$W(t,q,p) \equiv \frac{1}{2\pi}\int_{-\infty}^{+\infty} dy \varrho(t, q - y/2, q + y/2) e^{ipy},$$

- For pure states $|\psi\rangle$

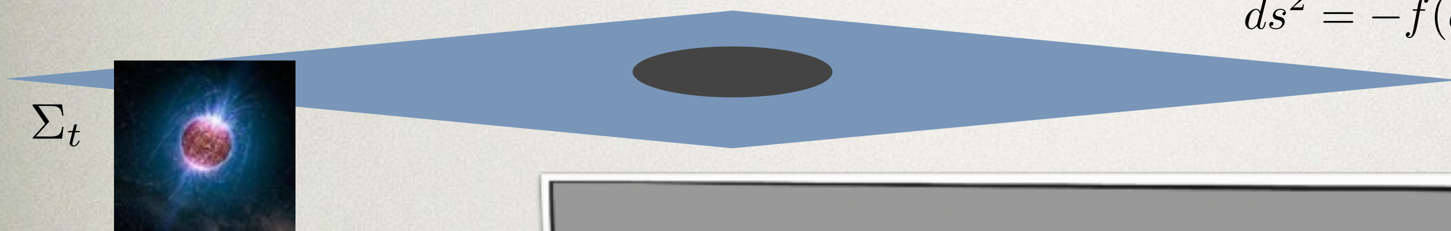
$$\int_{-\infty}^{+\infty} dq W(q,p) = |\tilde{\psi}(p)|^2, \; \int_{-\infty}^{+\infty} dp W(q,p) = |\psi(q)|^2$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t



It is a very useful tool to analyze the classicalization of a quantum system

- Wigner Function

$$W(t,q,p) \equiv \frac{1}{2\pi}\int_{-\infty}^{+\infty} dy \varrho(t, q - y/2, q + y/2) e^{ipy},$$

- For pure states $|\psi\rangle$

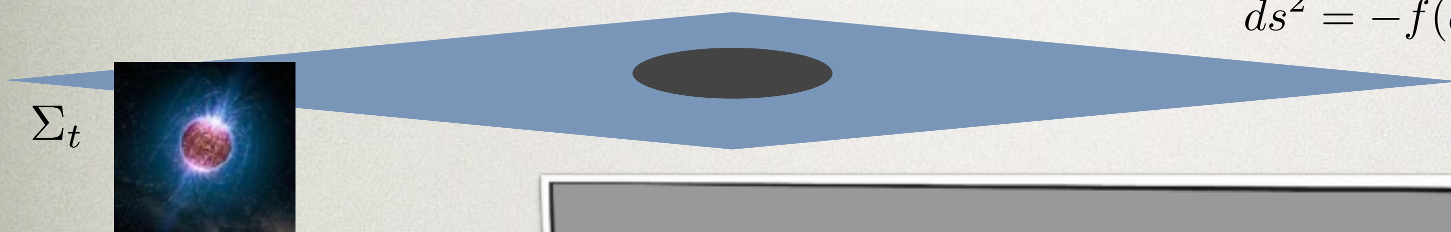
$$\int_{-\infty}^{+\infty} dq W(q,p) = |\tilde{\psi}(p)|^2, \int_{-\infty}^{+\infty} dp W(q,p) = |\psi(q)|^2$$

- For quadratic potentials (like our upside down harmonic oscillator)

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



It is a very useful tool to analyze the classicalization of a quantum system

- Wigner Function

$$W(t,q,p) \equiv \frac{1}{2\pi}\int_{-\infty}^{+\infty} dy\varrho(t, q - y/2, q + y/2)e^{ipy},$$

- For pure states $|\psi\rangle$

$$\int_{-\infty}^{+\infty} dqW(q,p) = |\tilde{\psi}(p)|^2, \quad \int_{-\infty}^{+\infty} dpW(q,p) = |\psi(q)|^2$$

- For quadratic potentials (like our upside down harmonic oscillator)

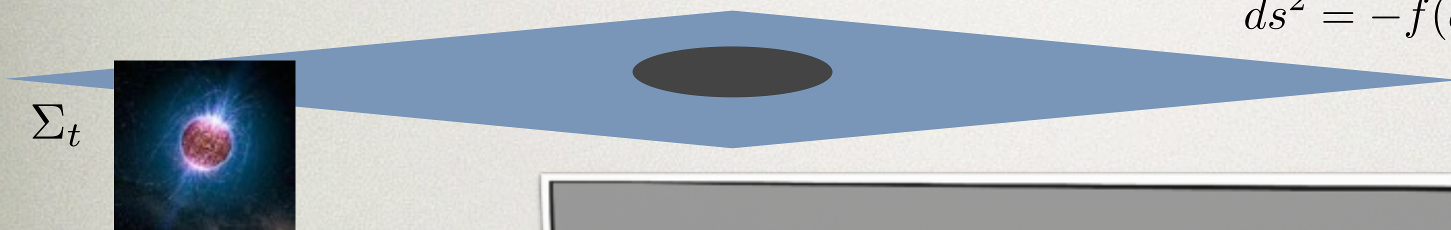
$$\partial_t W(t,q,p) = \{H(q,p), W(t,q,p)\}$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t



It is a very useful tool to analyze the classicalization of a quantum system

- Wigner Function

$$W(t,q,p) \equiv \frac{1}{2\pi}\int_{-\infty}^{+\infty} dy\varrho(t, q-y/2, q+y/2)e^{ipy},$$

- For pure states $|\psi\rangle$

$$\int_{-\infty}^{+\infty} dqW(q,p) = |\tilde{\psi}(p)|^2, \int_{-\infty}^{+\infty} dpW(q,p) = |\psi(q)|^2$$

- For quadratic potentials (like our upside down harmonic oscillator)

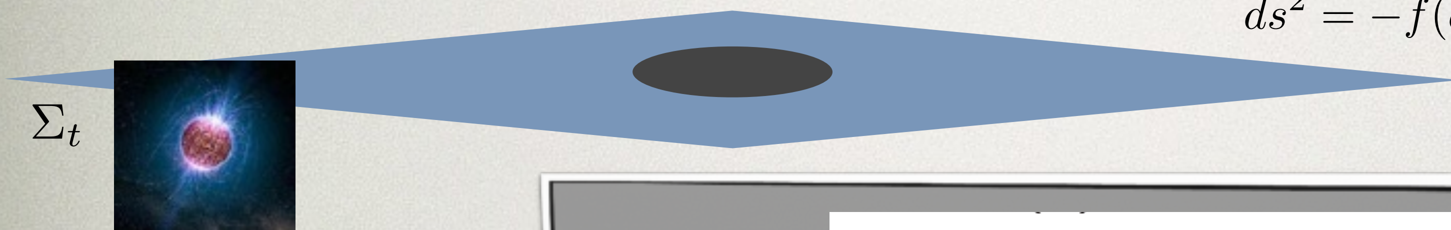
$$\partial_t W(t,q,p) = \{H(q,p), W(t,q,p)\}$$

Liouville equation

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



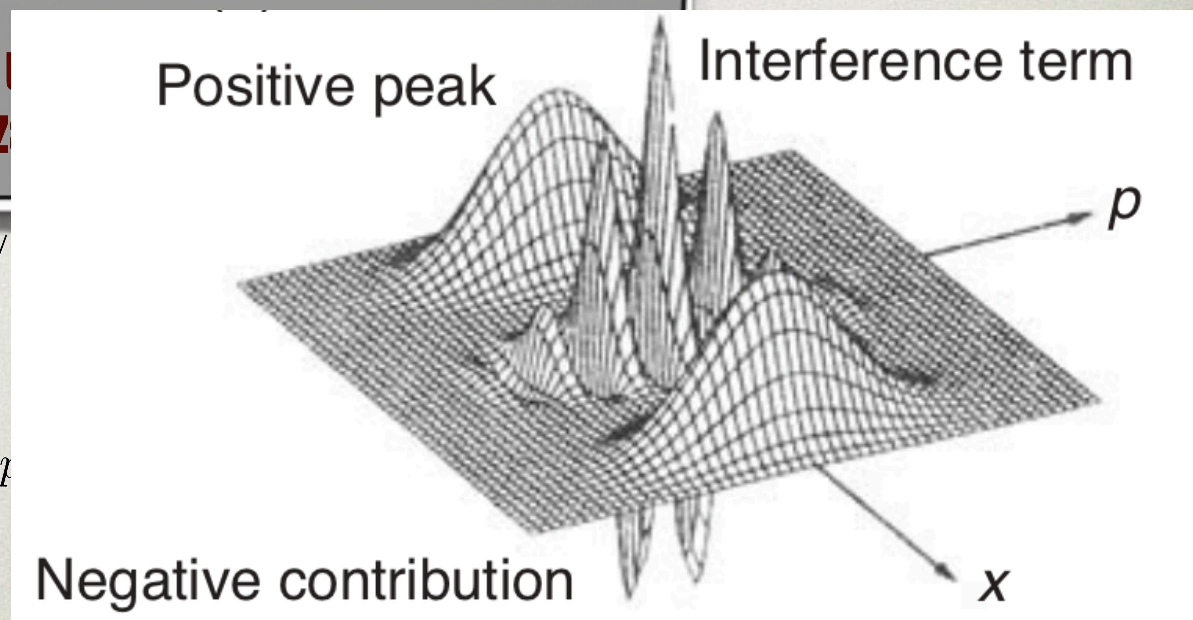
It is a very
classicaliz

- Wigner Function

$$W(t,q,p) \equiv \frac{1}{2\pi}\int_{-\infty}^{+\infty} dy \varrho(t, q - y/2, q + y/$$

- For pure states $|\psi\rangle$

$$\int_{-\infty}^{+\infty} dq W(q,p) = |\tilde{\psi}(p)|^2, \quad \int_{-\infty}^{+\infty} dp W(q,$$



W. Zurek, Decoherence and the Transition from Quantum to Classical—Revisited

- For quadratic potentials (like our ups

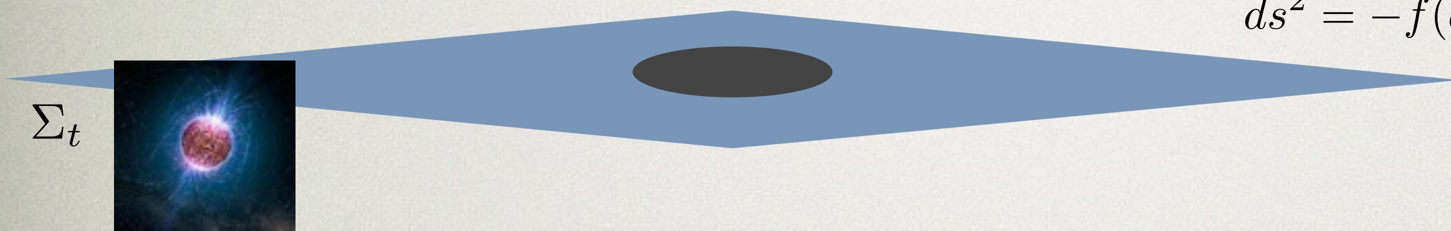
$$\partial_t W(t,q,p) = \{H(q,p), W(t,q,p)\}$$

Liouville equation

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

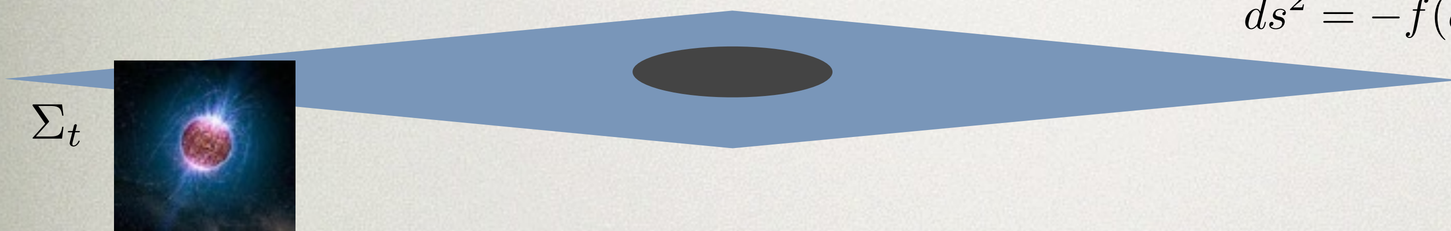


- For the upside down harmonic oscillator

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



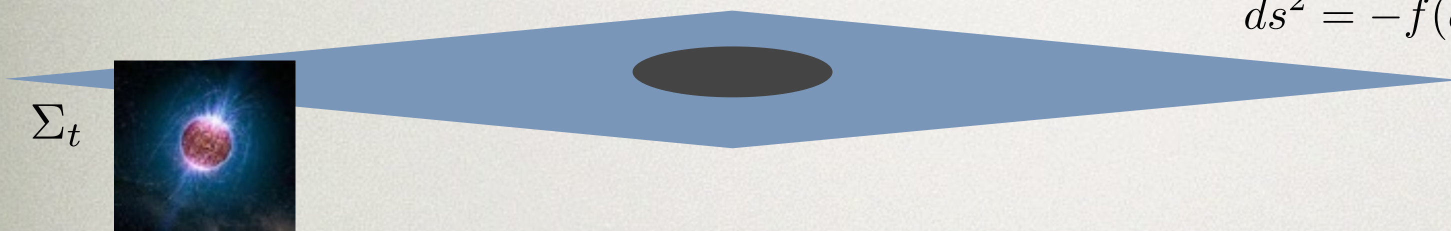
- For the upside down harmonic oscillator

$$H(q,p) \equiv \frac{1}{2}p^2 - \frac{\Omega^2}{2}q^2.$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Σ_t

- For the upside down harmonic oscillator

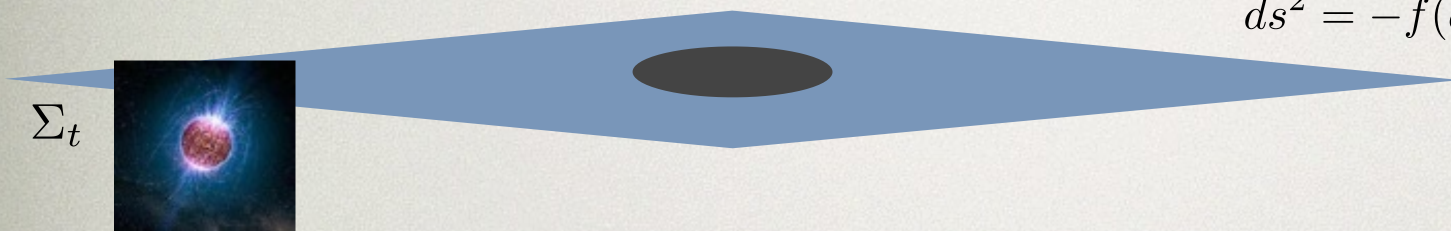
$$H(q,p) \equiv \frac{1}{2}p^2 - \frac{\Omega^2}{2}q^2.$$

$$\partial_t W = \Omega(u\partial_u - v\partial_v)W$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



- For the upside down harmonic oscillator

$$H(q,p) \equiv \frac{1}{2}p^2 - \frac{\Omega^2}{2}q^2.$$

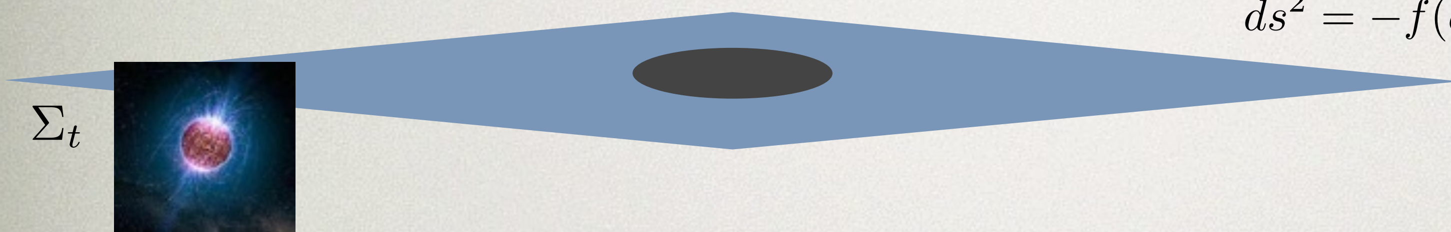
$$\partial_t W = \Omega(u\partial_u - v\partial_v)W$$

$$u \equiv \frac{p - \Omega q}{2} \qquad v \equiv \frac{p + \Omega q}{2}$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



- For the upside down harmonic oscillator

$$H(q,p) \equiv \frac{1}{2}p^2 - \frac{\Omega^2}{2}q^2.$$

$$\partial_t W = \Omega(u\partial_u - v\partial_v)W$$

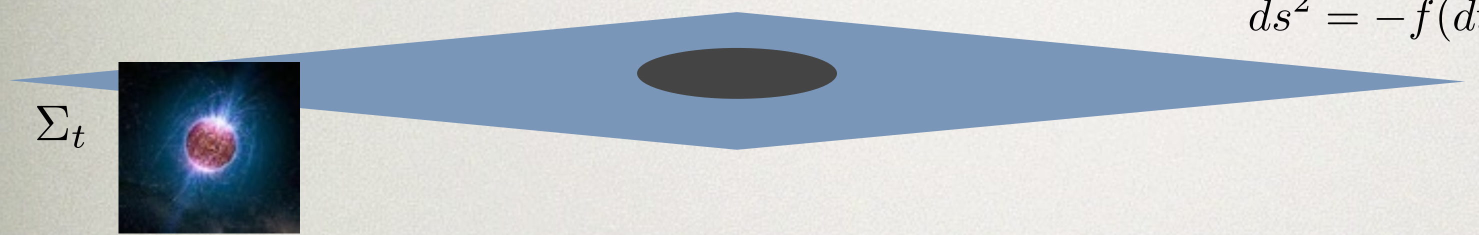
$$u \equiv \frac{p - \Omega q}{2} \qquad v \equiv \frac{p + \Omega q}{2}$$

- Squeezed vacuum

FROM QUANTUM TO CLASSICAL

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



- For the upside down harmonic oscillator

$$H(q,p) \equiv \frac{1}{2}p^2 - \frac{\Omega^2}{2}q^2.$$

$$\partial_t W = \Omega(u\partial_u - v\partial_v)W$$

$$u \equiv \frac{p - \Omega q}{2} \qquad v \equiv \frac{p + \Omega q}{2}$$

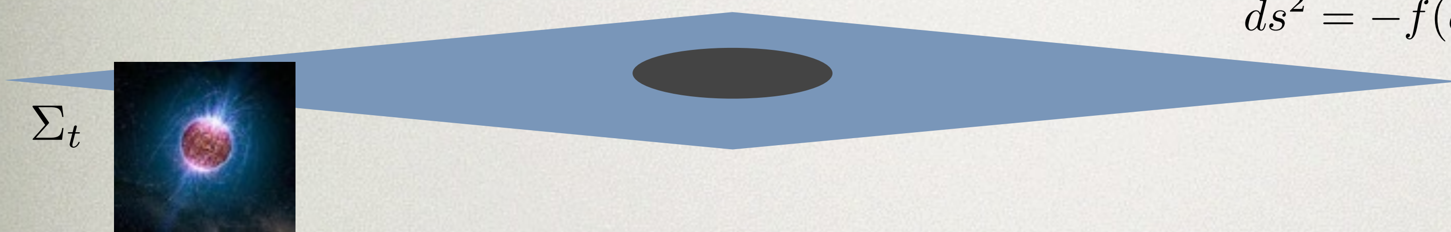
- Squeezed vacuum

$$\eta(t,q) \equiv \langle q|\eta(t)\rangle = \left(\frac{a(t)}{\pi}\right)^{1/4} e^{i\alpha(t)q^2/2}.$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



- For the upside down harmonic oscillator

$$H(q,p) \equiv \frac{1}{2}p^2 - \frac{\Omega^2}{2}q^2.$$

$$\partial_t W = \Omega(u\partial_u - v\partial_v)W$$

$$u \equiv \frac{p - \Omega q}{2} \qquad v \equiv \frac{p + \Omega q}{2}$$

- Squeezed vacuum

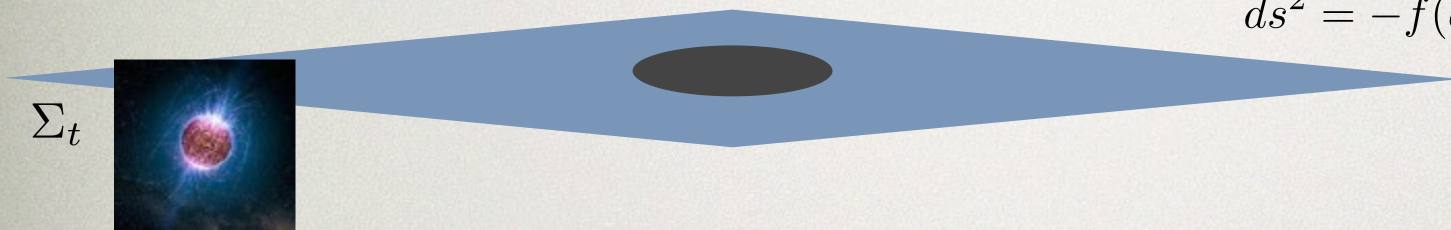
$$\eta(t,q) \equiv \langle q|\eta(t)\rangle = \left(\frac{a(t)}{\pi}\right)^{1/4} e^{i\alpha(t)q^2/2}.$$

$$W(t,q,p) = \frac{1}{\pi}\exp\left(-a(t)q^2 - \frac{[p+b(t)q]^2}{a(t)}\right).$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



- For the upside down harmonic oscillator

$$H(q,p) \equiv \frac{1}{2}p^2 - \frac{\Omega^2}{2}q^2.$$

$$\partial_t W = \Omega(u\partial_u - v\partial_v)W$$

$$u \equiv \frac{p - \Omega q}{2} \qquad v \equiv \frac{p + \Omega q}{2}$$

- Squeezed vacuum

$$\eta(t,q) \equiv \langle q|\eta(t)\rangle = \left(\frac{a(t)}{\pi}\right)^{1/4} e^{i\alpha(t)q^2/2}.$$

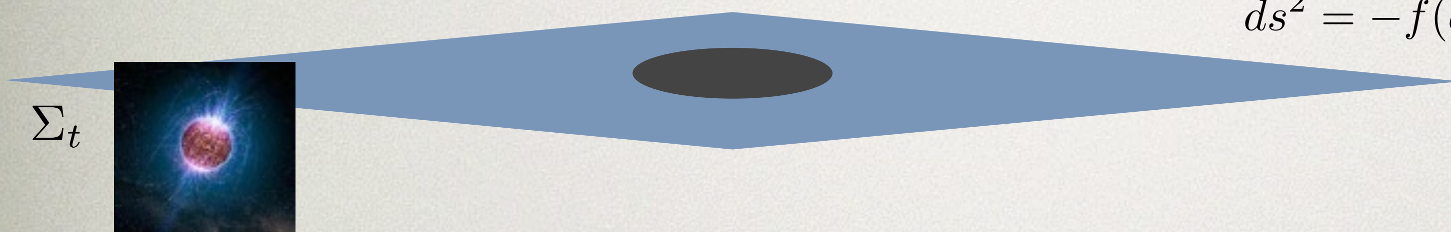
$$W(t,q,p) = \frac{1}{\pi}\exp\left(-a(t)q^2 - \frac{[p + b(t)q]^2}{a(t)}\right).$$

$$W(t,q,p) \overset{\Omega t \gg 1}{\approx} |\eta(t,q)|^2 \delta(p - \Omega q),$$

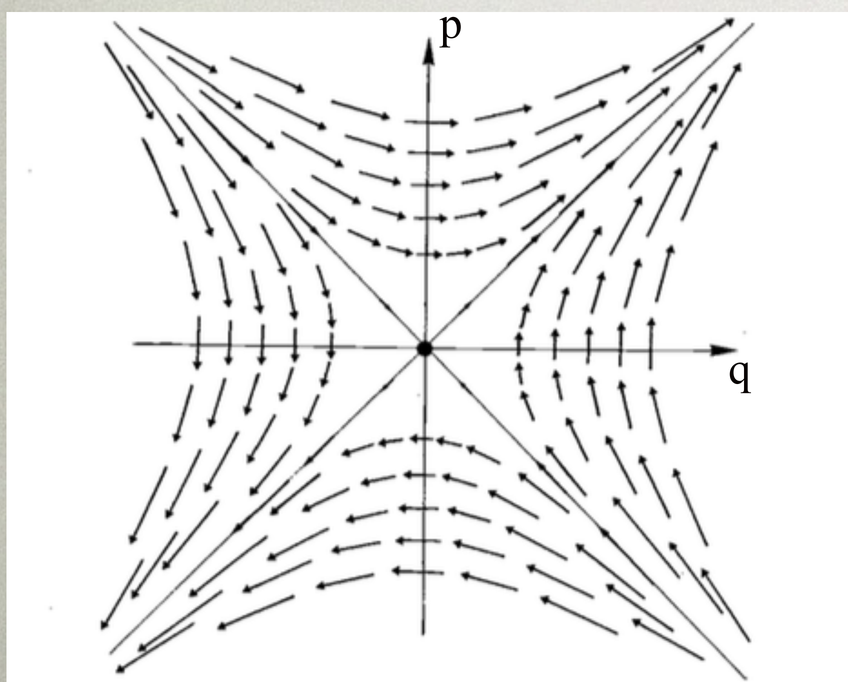
From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



- For the upside down harmonic oscillator



- Squeezed vacuum

$$\eta(t,q) \equiv \langle q|\eta(t)\rangle = \left(\frac{a(t)}{\pi}\right)^{1/4} e^{i\alpha(t)q^2/2}.$$

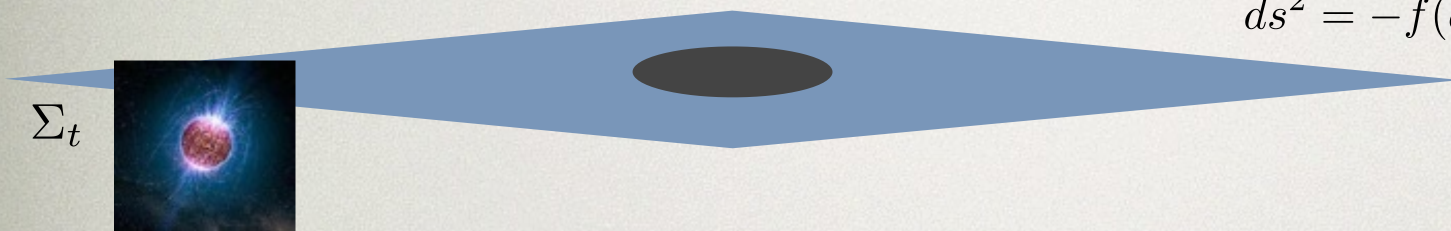
$$W(t,q,p) = \frac{1}{\pi}\exp\left(-a(t)q^2 - \frac{[p+b(t)q]^2}{a(t)}\right).$$

$$W(t,q,p) \overset{\Omega t \gg 1}{\approx} |\eta(t,q)|^2 \delta(p - \Omega q),$$

From quantum to classical

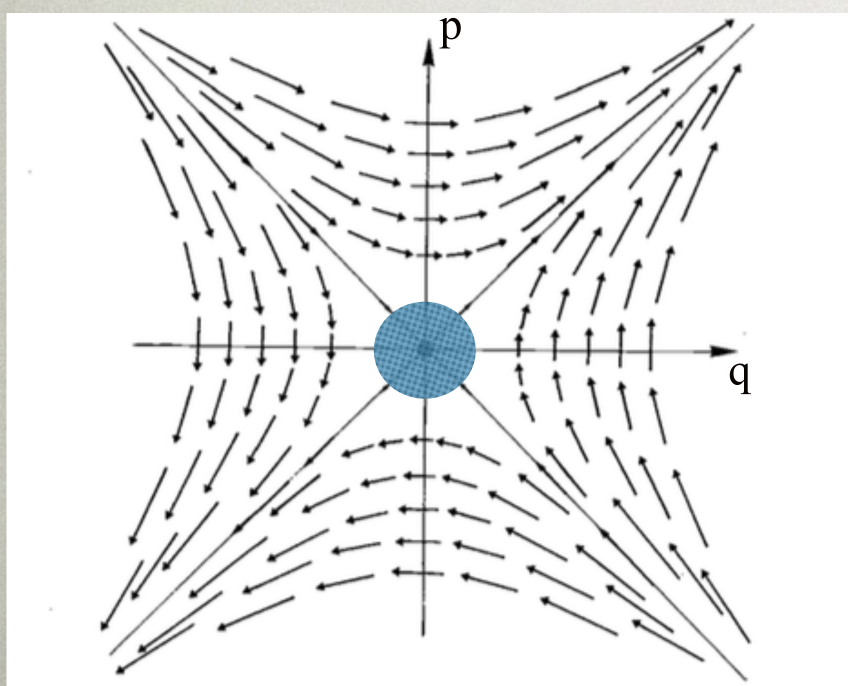
- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Σ_t

- For the upside down harmonic oscillator



- Squeezed vacuum

$$\eta(t,q) \equiv \langle q|\eta(t)\rangle = \left(\frac{a(t)}{\pi}\right)^{1/4} e^{i\alpha(t)q^2/2}.$$

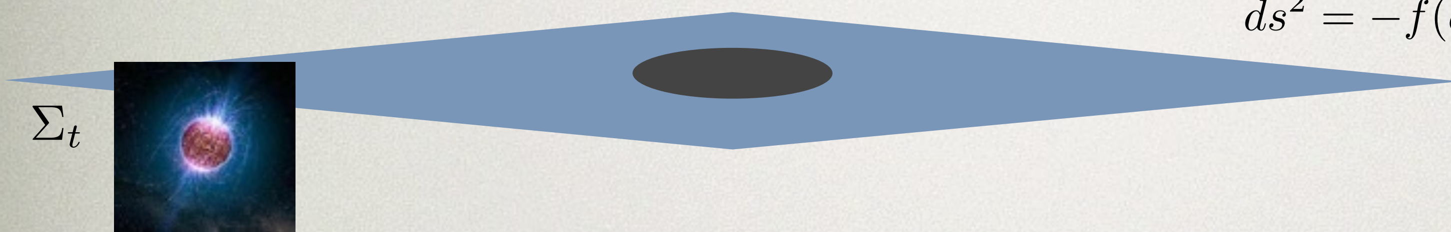
$$W(t,q,p) = \frac{1}{\pi}\exp\left(-a(t)q^2 - \frac{[p+b(t)q]^2}{a(t)}\right).$$

$$W(t,q,p) \overset{\Omega t \gg 1}{\approx} |\eta(t,q)|^2 \delta(p - \Omega q),$$

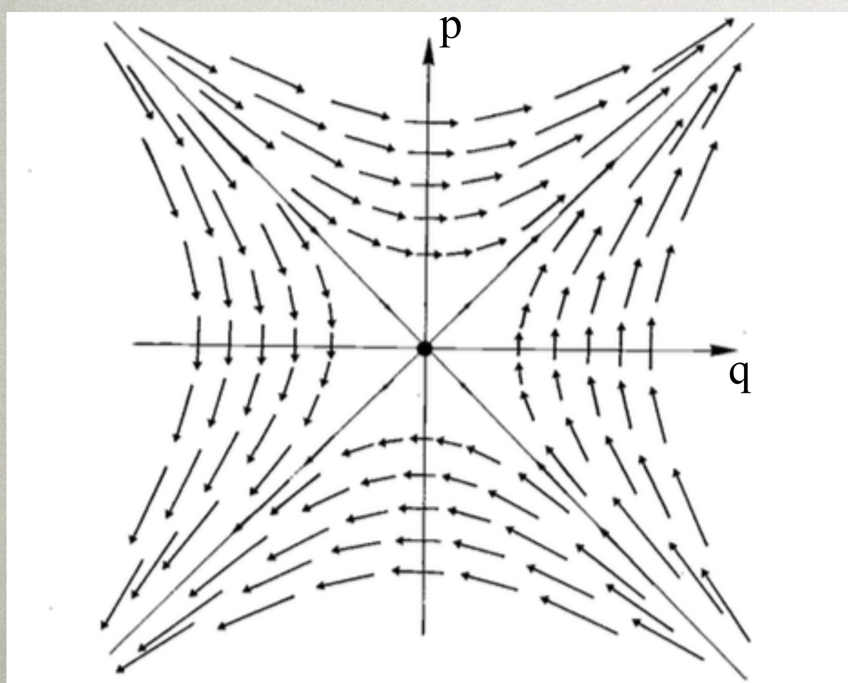
From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



- For the upside down harmonic oscillator



- Squeezed vacuum

$$\eta(t,q) \equiv \langle q|\eta(t)\rangle = \left(\frac{a(t)}{\pi}\right)^{1/4} e^{i\alpha(t)q^2/2}.$$

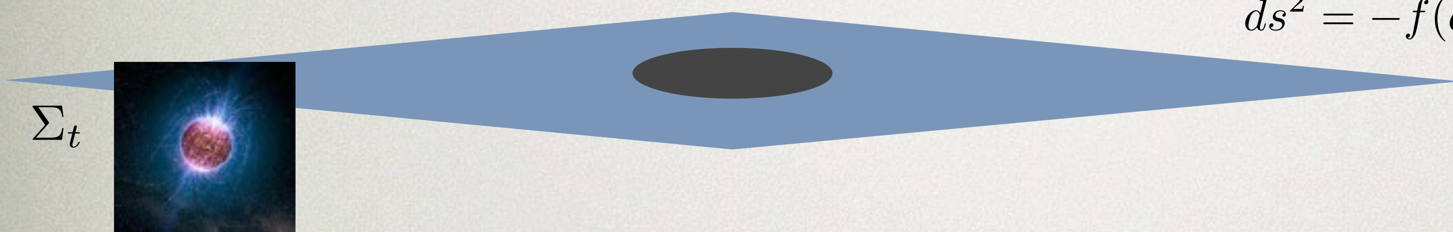
$$W(t,q,p) = \frac{1}{\pi}\exp\left(-a(t)q^2 - \frac{[p+b(t)q]^2}{a(t)}\right).$$

$$W(t,q,p) \overset{\Omega t \gg 1}{\approx} |\eta(t,q)|^2 \delta(p - \Omega q),$$

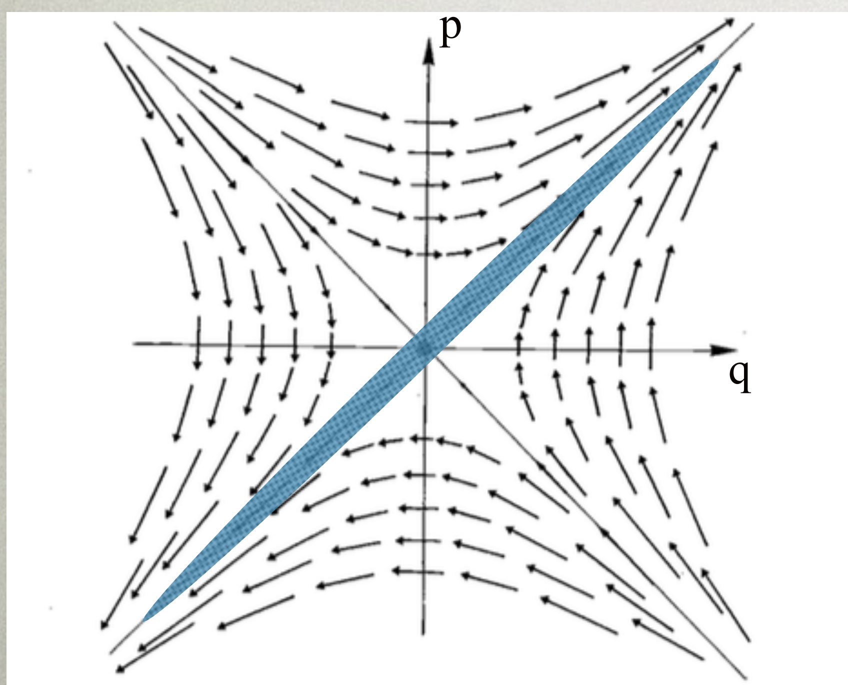
From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



- For the upside down harmonic oscillator



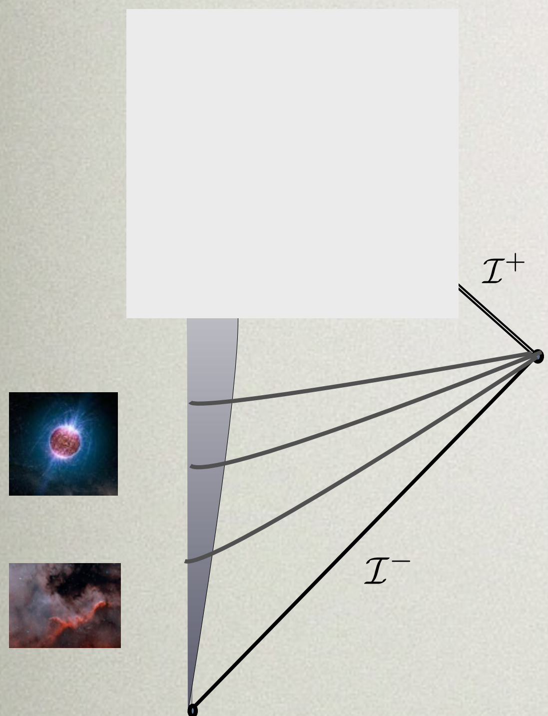
- Squeezed vacuum

$$\eta(t,q) \equiv \langle q|\eta(t)\rangle = \left(\frac{a(t)}{\pi}\right)^{1/4} e^{i\alpha(t)q^2/2}.$$

$$W(t,q,p) = \frac{1}{\pi}\exp\left(-a(t)q^2 - \frac{[p+b(t)q]^2}{a(t)}\right).$$

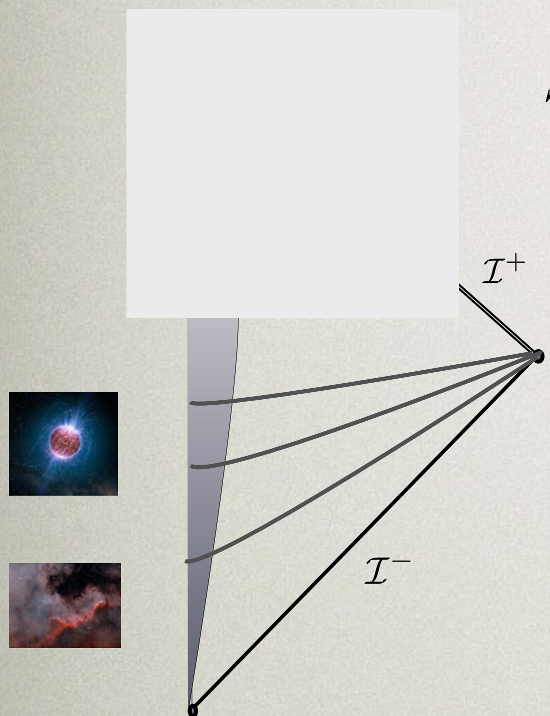
$$W(t,q,p) \overset{\Omega t \gg 1}{\approx} |\eta(t,q)|^2\delta(p-\Omega q),$$

From quantum to classical



From quantum to classical

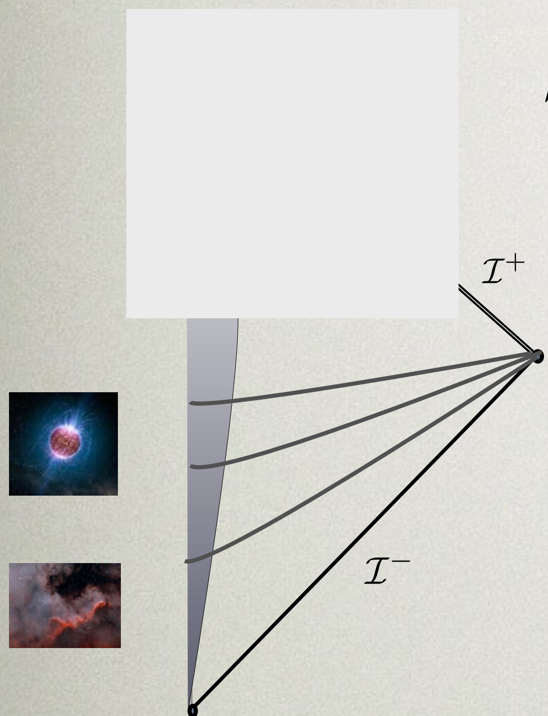
$$S = S_{\mathrm{EH}} + S_{\Phi} + S_{\mathrm{M}}$$



From quantum to classical

$$S = S_{\mathrm{EH}} + S_{\Phi} + S_{\mathrm{M}}$$

$$S_{\mathrm{EH}}[g_{ab}] \equiv \frac{2}{\kappa^2}\int_{\mathcal{M}} d^4x\sqrt{-g}R, \quad \kappa \equiv \sqrt{32\pi G}$$

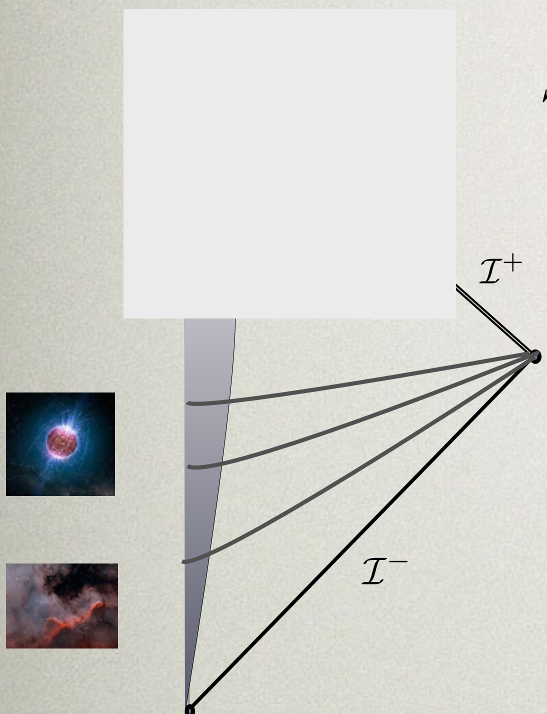


From quantum to classical

$$S = S_{\mathrm{EH}} + S_{\Phi} + S_{\mathrm{M}}$$

$$S_{\mathrm{EH}}[g_{ab}] \equiv \frac{2}{\kappa^2}\int_{\mathcal{M}} d^4x\sqrt{-g}R, \quad \kappa \equiv \sqrt{32\pi G}$$

Einstein-Hilbert action



From quantum to classical

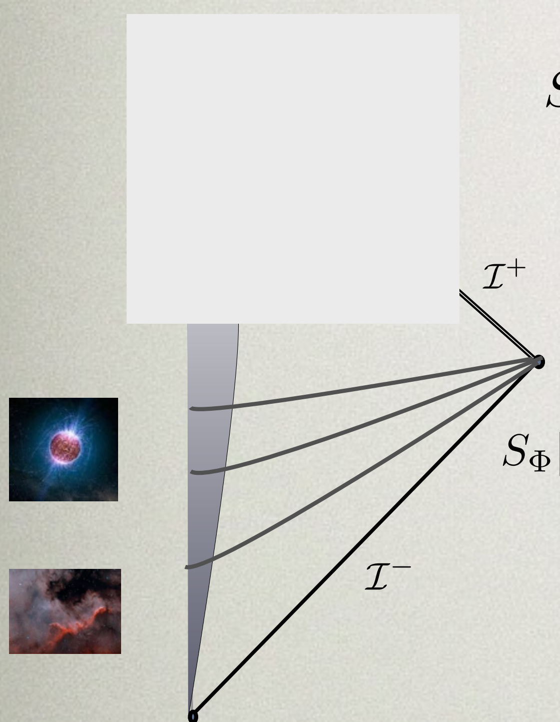
$$S = S_{\rm EH} + S_\Phi + S_{\rm M}$$

$$S_{\rm EH}[g_{ab}] \equiv \frac{2}{\kappa^2}\int_{\mathcal{M}} d^4x\sqrt{-g}R, \quad \kappa \equiv \sqrt{32\pi G}$$

Einstein-Hilbert action

$$S_\Phi[\Phi, g_{ab}] \equiv -\frac{1}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}[\nabla_a\Phi\nabla^a\Phi + (m^2+\xi R)\Phi^2],$$

From quantum to classical



$$S = S_{\mathrm{EH}} + S_{\Phi} + S_{\mathrm{M}}$$

$$S_{\mathrm{EH}}[g_{ab}] \equiv \frac{2}{\kappa^2}\int_{\mathcal{M}} d^4x\sqrt{-g}R, \quad \kappa \equiv \sqrt{32\pi G}$$

Einstein-Hilbert action

$$S_{\Phi}[\Phi, g_{ab}] \equiv -\frac{1}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}[\nabla_a\Phi\nabla^a\Phi + (m^2+\xi R)\Phi^2],$$

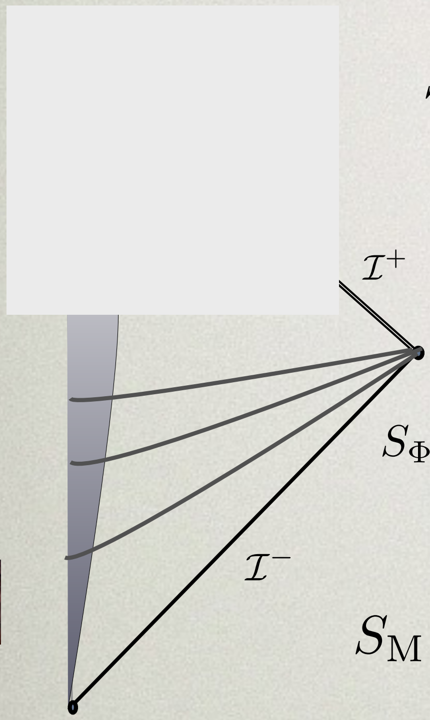
scalar field action

From quantum to classical

$$S = S_{\rm EH} + S_\Phi + S_{\rm M}$$

$$S_{\rm EH}[g_{ab}] \equiv \frac{2}{\kappa^2}\int_{\mathcal{M}} d^4x\sqrt{-g}R, \quad \kappa \equiv \sqrt{32\pi G}$$

Einstein-Hilbert action

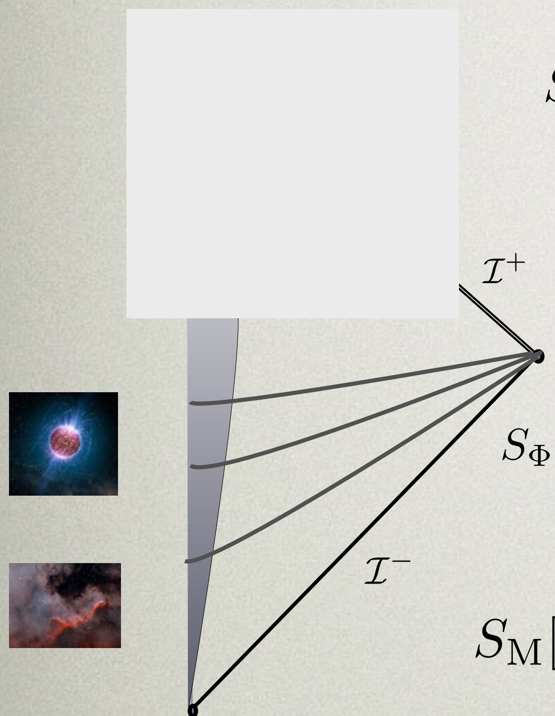


$$S_\Phi[\Phi, g_{ab}] \equiv -\frac{1}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}[\nabla_a\Phi\nabla^a\Phi + (m^2+\xi R)\Phi^2],$$

scalar field action

$$S_{\rm M}[\Psi, g_{ab}]$$

From quantum to classical



$$S = S_{\mathrm{EH}} + S_{\Phi} + S_{\mathrm{M}}$$

$$S_{\mathrm{EH}}[g_{ab}] \equiv \frac{2}{\kappa^2}\int_{\mathcal{M}} d^4x\sqrt{-g}R, \quad \kappa \equiv \sqrt{32\pi G}$$

Einstein-Hilbert action

$$S_{\Phi}[\Phi, g_{ab}] \equiv -\frac{1}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}[\nabla_a\Phi\nabla^a\Phi + (m^2+\xi R)\Phi^2],$$

scalar field action

$$S_{\mathrm{M}}[\Psi, g_{ab}]$$

classical-matter action

From quantum to classical

$$S = S_{\rm EH} + S_\Phi + S_{\rm M}$$

$$S_{\rm EH}[g_{ab}] \equiv \frac{2}{\kappa^2}\int_{\mathcal{M}} d^4x\sqrt{-g}R, \quad \kappa \equiv \sqrt{32\pi G}$$

Einstein-Hilbert action

$$S_\Phi[\Phi, g_{ab}] \equiv -\frac{1}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}[\nabla_a\Phi\nabla^a\Phi + (m^2+\xi R)\Phi^2],$$

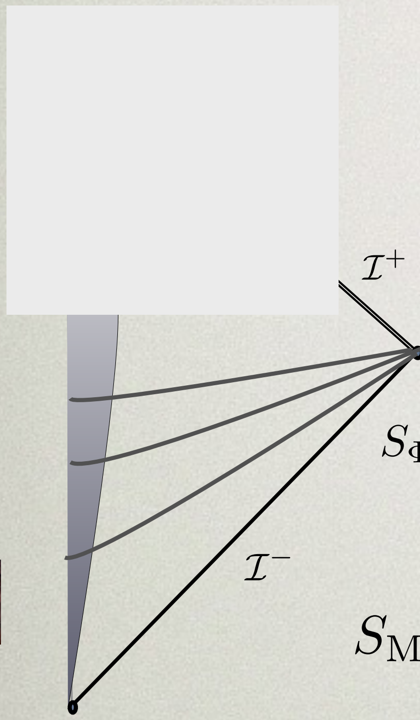
scalar field action

$$S_{\rm M}[\Psi, g_{ab}]$$

classical-matter action

$$g_{ab} \to g_{ab} + \kappa\gamma_{ab}, \Phi \to \Phi + \phi \; (\Phi = 0)$$

From quantum to classical



$$S = S_{\rm EH} + S_\Phi + S_{\rm M}$$

$$S_{\rm EH}[g_{ab}] \equiv \frac{2}{\kappa^2}\int_{\mathcal{M}} d^4x\sqrt{-g}R, \quad \kappa \equiv \sqrt{32\pi G}$$

Einstein-Hilbert action

$$S_\Phi[\Phi, g_{ab}] \equiv -\frac{1}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}[\nabla_a\Phi\nabla^a\Phi + (m^2+\xi R)\Phi^2],$$

scalar field action

$$S_{\rm M}[\Psi, g_{ab}]$$

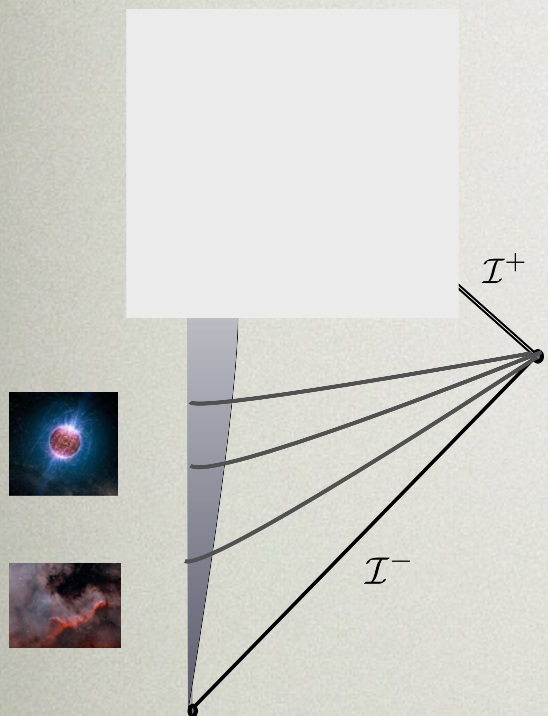
classical-matter action

$$g_{ab} \to g_{ab} + \kappa\gamma_{ab}, \Phi \to \Phi + \phi \; (\Phi = 0)$$

Action describing the perturbations and its interaction

$$S_\phi[\phi] + S_\gamma[\gamma_{ab}] + S_{\rm int}[\phi, \gamma_{ab}]$$

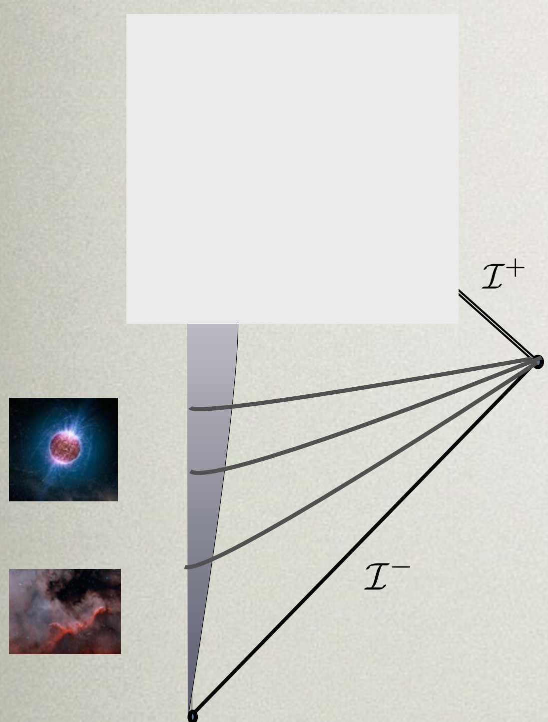
From quantum to classical



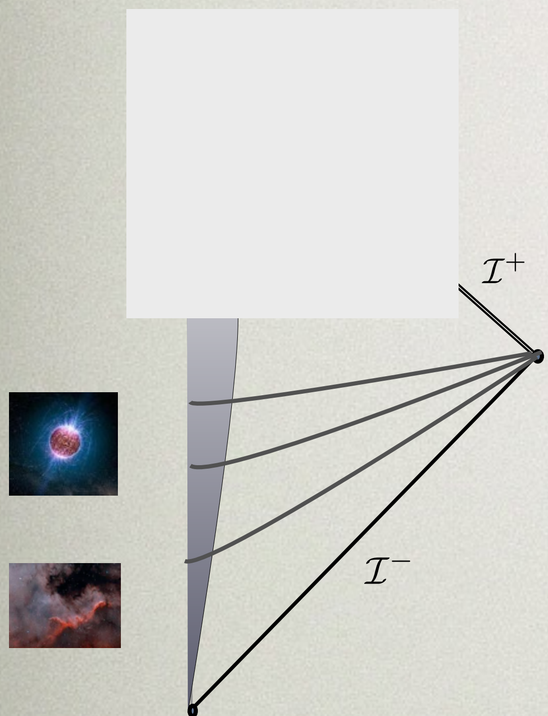
From quantum to classical

free scalar field

$$S_\phi[\phi] \equiv -\frac{1}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}\phi P\phi,\ P \equiv -(\nabla^a\nabla_a + m^2 + \xi R)$$



From quantum to classical



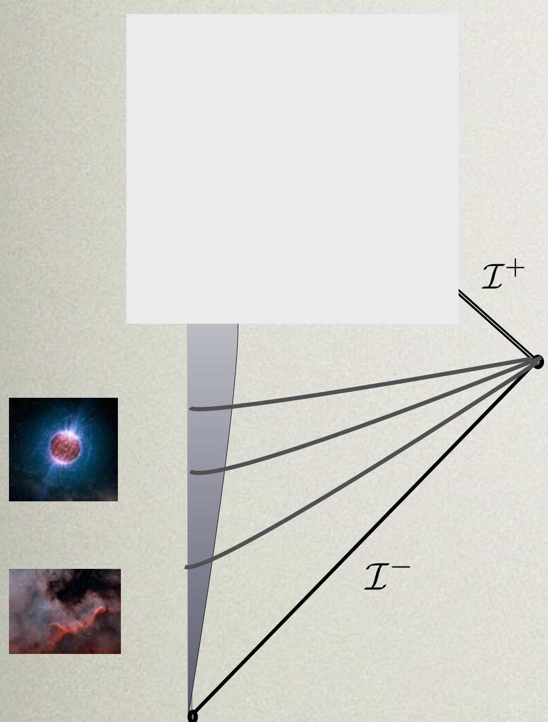
free scalar field

$$S_\phi[\phi] \equiv -\frac{1}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}\phi P\phi,\ P \equiv -(\nabla^a\nabla_a + m^2 + \xi R)$$

free graviton (with gauge fixing term)

$$S_\gamma[\gamma_{ab}] \equiv -\frac{1}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}\gamma^{ab}\mathfrak{D}_{abcd}\gamma^{cd}$$

From quantum to classical



free scalar field

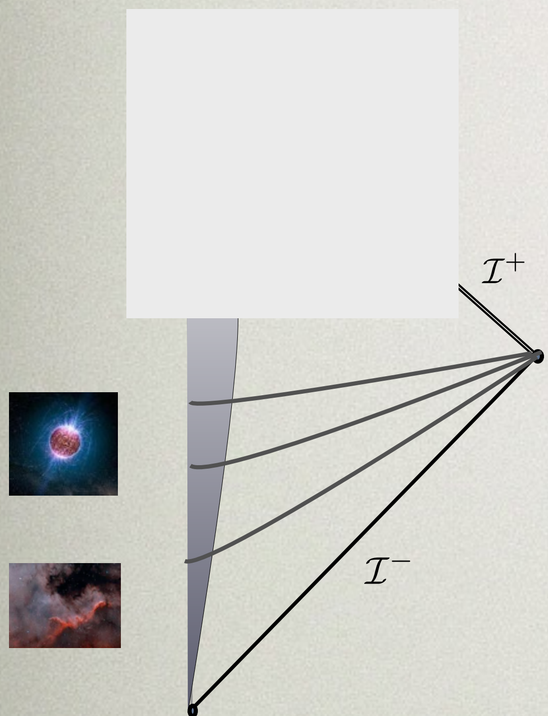
$$S_\phi[\phi] \equiv -\frac{1}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}\phi P\phi,\ P \equiv -(\nabla^a\nabla_a + m^2 + \xi R)$$

free graviton (with gauge fixing term)

$$S_\gamma[\gamma_{ab}] \equiv -\frac{1}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}\gamma^{ab}\mathfrak{D}_{abcd}\gamma^{cd}$$

$$\hat{\gamma}_{ab}(t,\mathbf{x}) = \sum_j \int \frac{d\vartheta(\alpha)}{\sqrt{2\omega_\alpha}}\hat{b}_\alpha^{(j)} e^{-i\omega_\alpha t}\varepsilon_{\alpha ab}^{(j)}(\mathbf{x}) + \mathrm{H.c.}$$

From quantum to classical



free scalar field

$$S_\phi[\phi] \equiv -\frac{1}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}\phi P\phi,\ P \equiv -(\nabla^a\nabla_a + m^2 + \xi R)$$

free graviton (with gauge fixing term)

$$S_\gamma[\gamma_{ab}] \equiv -\frac{1}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}\gamma^{ab}\mathfrak{D}_{abcd}\gamma^{cd}$$

$$\hat{\gamma}_{ab}(t,\mathbf{x}) = \sum_j \int \frac{d\vartheta(\alpha)}{\sqrt{2\omega_\alpha}}\hat{b}^{(j)}_\alpha e^{-i\omega_\alpha t}\varepsilon^{(j)}_{\alpha ab}(\mathbf{x}) + \mathrm{H.c.}$$

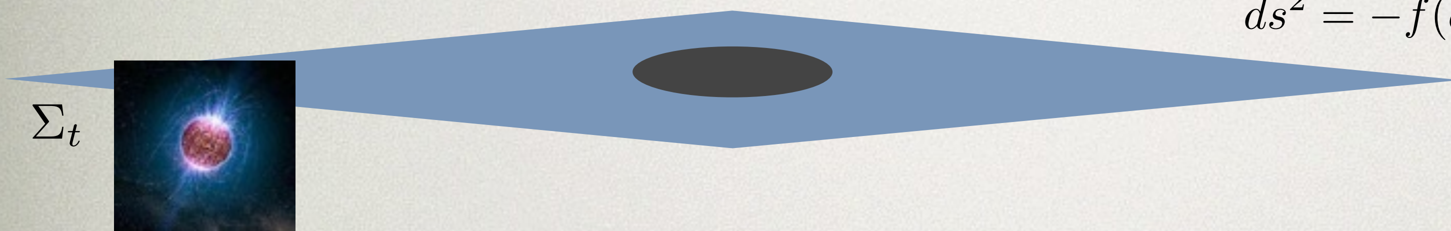
Interaction

$$S_{\mathrm{int}}[\phi,\gamma_{ab}] \equiv \int_{\mathcal{M}} d^4x\sqrt{-g}[\mathcal{L}^{(3)}_\Phi + \mathcal{L}^{(4)}_\Phi] \qquad \mathcal{L}^{(3)}_\Phi \equiv \frac{\kappa}{2}T_{ab}\gamma^{ab}$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

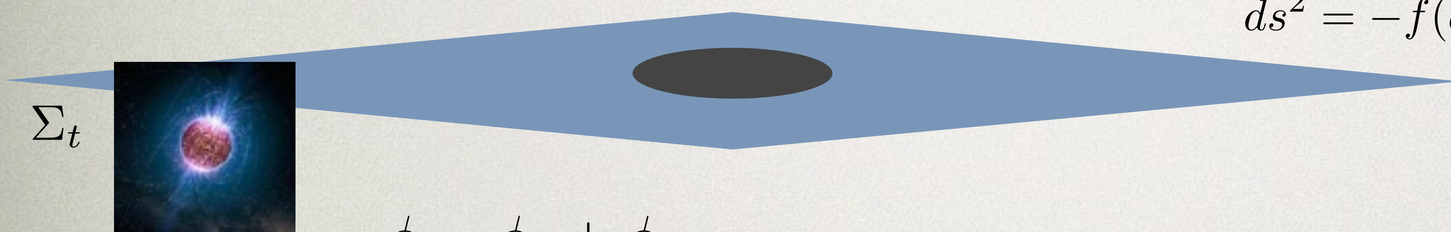


From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t

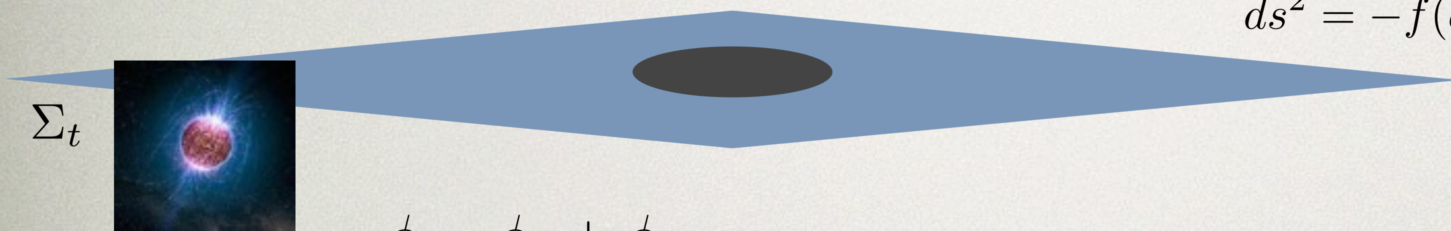


$$\phi = \phi_s + \phi_u$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Σ_t

$$\phi = \phi_s + \phi_u$$

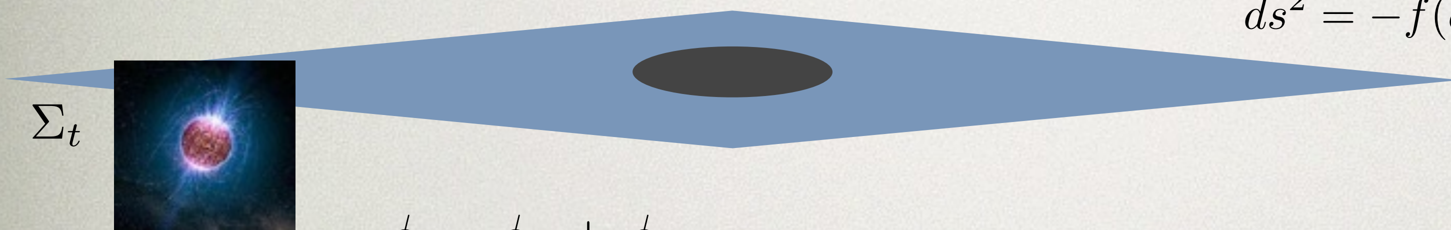
System of interest - Unstable mode (which is the one we expect that "classicalize")

$$\phi_u$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Σ_t

$$\phi = \phi_s + \phi_u$$

System of interest - Unstable mode (which is the one we expect that "classicalize")

ϕ_u

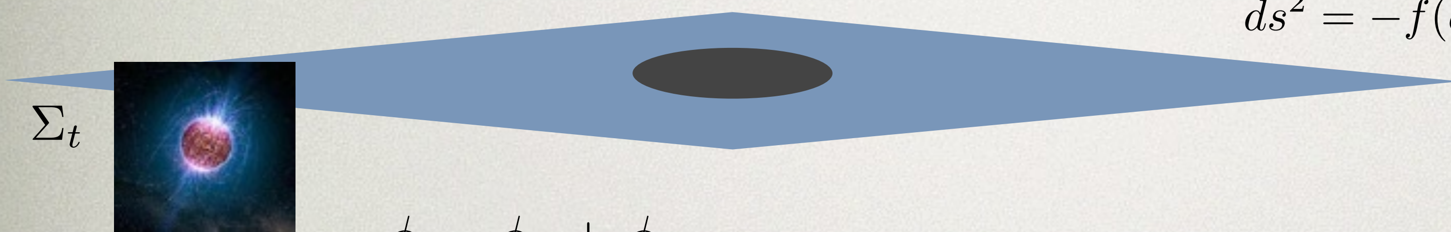
Environment - Stable modes and graviton

$\phi_s, \ \gamma_{ab}$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Σ_t

$$\phi = \phi_s + \phi_u$$

System of interest - Unstable mode (which is the one we expect that "classicalize")

$$\phi_u$$

Environment - Stable modes and graviton

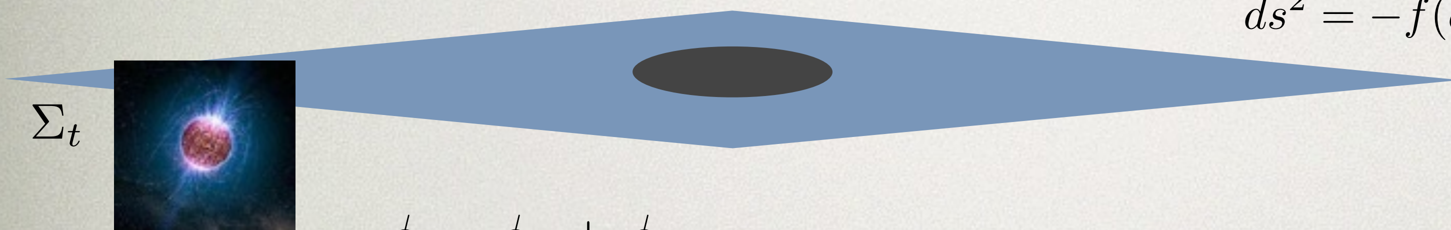
$$\phi_s,\ \gamma_{ab}$$

$$S_\phi[\phi] = S_\phi[\phi_\mathrm{s}] + S_\phi[\phi_\mathrm{u}]$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Σ_t

$$\phi = \phi_s + \phi_u$$

System of interest - Unstable mode (which is the one we expect that "classicalize")

$$\phi_u$$

$$S_\phi[\phi] = S_\phi[\phi_s] + S_\phi[\phi_u]$$

Environment - Stable modes and graviton

$$\phi_s, \ \gamma_{ab}$$

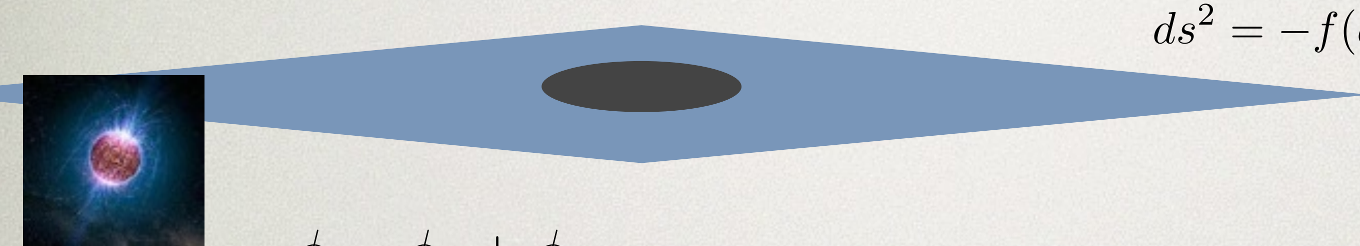
$$T_{ab} = T_{ab}^{(s)} + T_{ab}^{(u)} + t_{ab}$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t



$$\phi = \phi_s + \phi_u$$

System of interest - Unstable mode (which is the one we expect that "classicalize")

$$\phi_u$$

Environment - Stable modes and graviton

$$\phi_s,\ \gamma_{ab}$$

$$S_\phi[\phi] = S_\phi[\phi_{\rm s}] + S_\phi[\phi_{\rm u}]$$

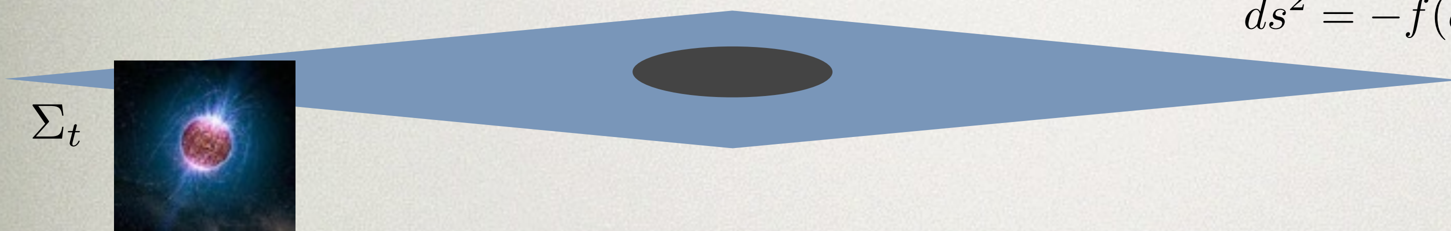
$$T_{ab} = T_{ab}^{(\rm s)} + T_{ab}^{(\rm u)} + t_{ab}$$

$$S_{\rm int}[\phi, \gamma_{ab}] \equiv \frac{\kappa}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}T_{ab}^{(s)}\gamma^{ab} + \frac{\kappa}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}T_{ab}^{(u)}\gamma^{ab} + \frac{\kappa}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}t_{ab}\gamma^{ab}$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



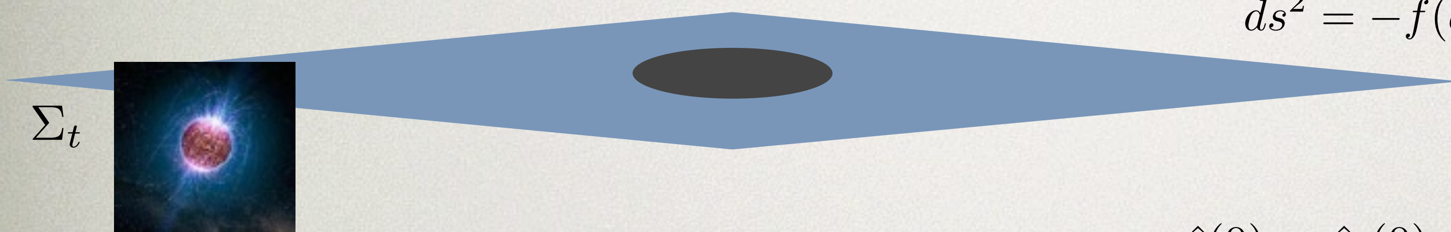
Σ_t

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t



$$\hat{\varrho}(0) = \hat{\varrho}_{\mathrm{s}}(0) \otimes \hat{\varrho}_{\mathrm{u}}(0) \otimes \hat{\varrho}_{\gamma}(0),$$

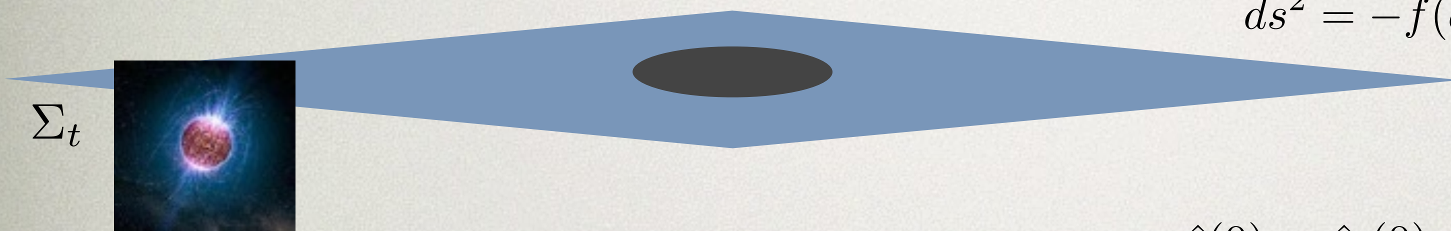
Initial state of the full system

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t



$$\hat{\varrho}(0) = \hat{\varrho}_{\rm s}(0) \otimes \hat{\varrho}_{\rm u}(0) \otimes \hat{\varrho}_{\gamma}(0),$$

Initial state of the full system

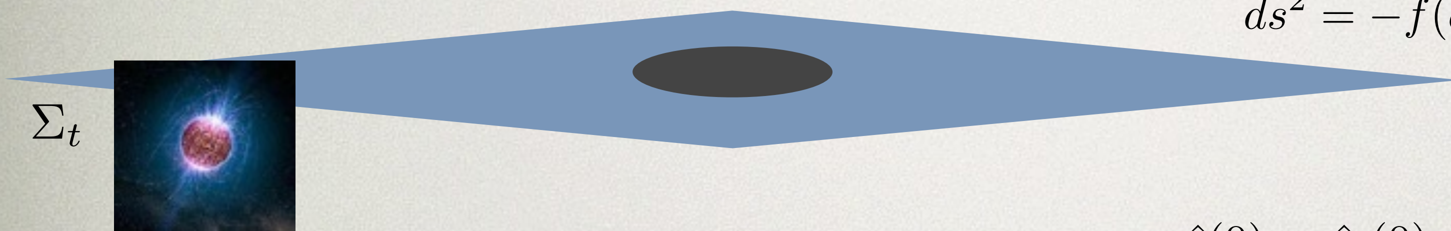
$$\hat{\varrho}_{\rm s}(0) = |0_{\rm s}\rangle\langle 0_{\rm s}|$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t



$$\hat{\varrho}(0) = \hat{\varrho}_{\rm s}(0) \otimes \hat{\varrho}_{\rm u}(0) \otimes \hat{\varrho}_\gamma(0),$$

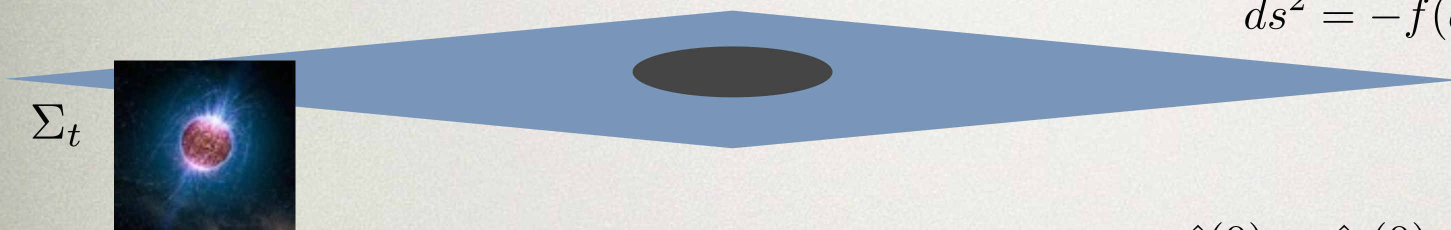
$$\hat{\varrho}_{\rm s}(0) = |0_{\rm s}\rangle\langle 0_{\rm s}| \qquad \hat{\varrho}_{\rm u}(0) = |0_{\rm u}\rangle\langle 0_{\rm u}|$$

Initial state of the full system

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Initial state of the full system

$$\hat{\varrho}(0) = \hat{\varrho}_{\rm s}(0) \otimes \hat{\varrho}_{\rm u}(0) \otimes \hat{\varrho}_{\gamma}(0),$$

$$\hat{\varrho}_{\rm s}(0) = |0_{\rm s}\rangle\langle 0_{\rm s}| \qquad \hat{\varrho}_{\rm u}(0) = |0_{\rm u}\rangle\langle 0_{\rm u}|$$

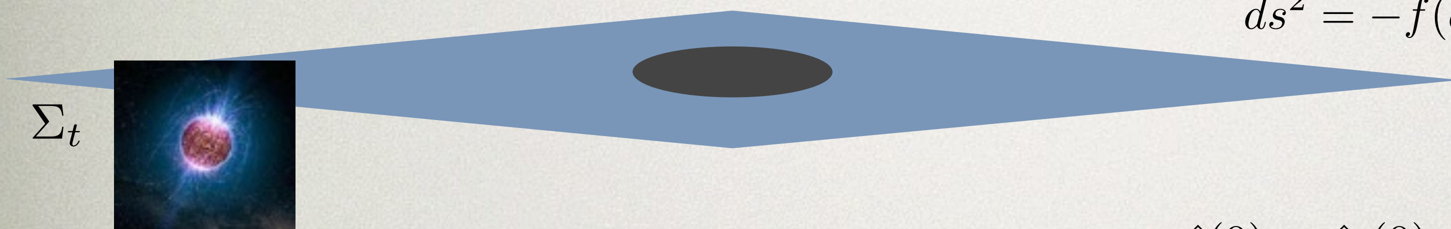
$\hat{\varrho}_{\gamma}(0) =$ thermal state with temperature T

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t



$$\hat{\varrho}(0) = \hat{\varrho}_{\mathrm{s}}(0) \otimes \hat{\varrho}_{\mathrm{u}}(0) \otimes \hat{\varrho}_{\gamma}(0),$$

Initial state of the full system

$$\hat{\varrho}_{\mathrm{s}}(0) = |0_{\mathrm{s}}\rangle\langle 0_{\mathrm{s}}| \qquad \hat{\varrho}_{\mathrm{u}}(0) = |0_{\mathrm{u}}\rangle\langle 0_{\mathrm{u}}|$$

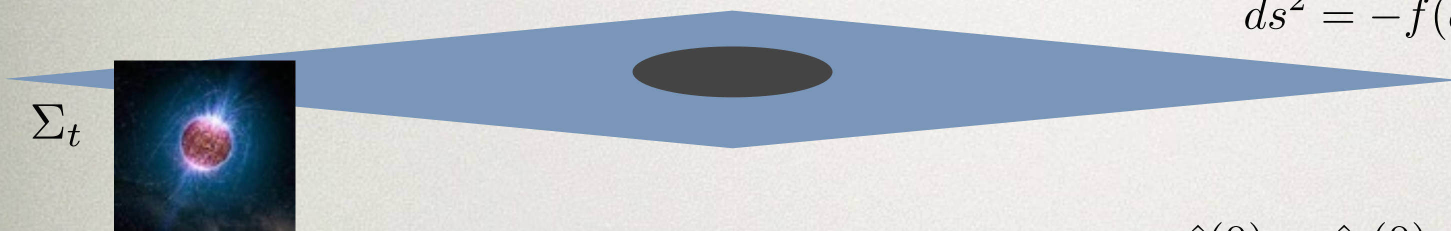
$\hat{\varrho}_{\gamma}(0) = \text{thermal state with temperature } T$

$$\hat{\rho}(0) \underset{\hat{U}_{\mathrm{u}\gamma\mathrm{s}}(t)}{\longrightarrow} \hat{\rho}(t)$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Initial state of the full system

$$\hat{\varrho}(0) = \hat{\varrho}_{\rm s}(0) \otimes \hat{\varrho}_{\rm u}(0) \otimes \hat{\varrho}_{\gamma}(0),$$

$$\hat{\varrho}_{\rm s}(0) = |0_{\rm s}\rangle\langle 0_{\rm s}| \qquad \hat{\varrho}_{\rm u}(0) = |0_{\rm u}\rangle\langle 0_{\rm u}|$$

$\hat{\varrho}_{\gamma}(0) =$ thermal state with temperature T

$$\hat{\rho}(0) \underset{\hat{U}_{{\rm u}\gamma{\rm s}}(t)}{\longrightarrow} \hat{\rho}(t) \xrightarrow{\hphantom{xxxxxxxx}}$$

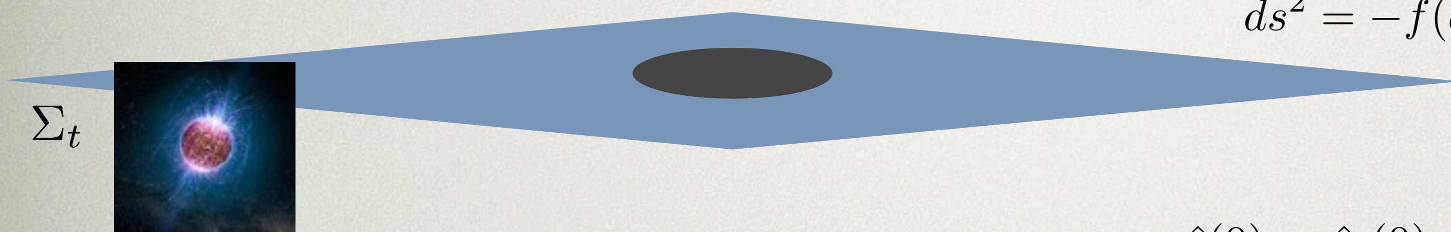
trace out the environment

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta \, d\varphi^2)$$

Σ_t



Initial state of the full system

$$\hat{\varrho}(0) = \hat{\varrho}_{\rm s}(0) \otimes \hat{\varrho}_{\rm u}(0) \otimes \hat{\varrho}_\gamma(0),$$

$$\hat{\varrho}_{\rm s}(0) = |0_{\rm s}\rangle\langle 0_{\rm s}| \qquad \hat{\varrho}_{\rm u}(0) = |0_{\rm u}\rangle\langle 0_{\rm u}|$$

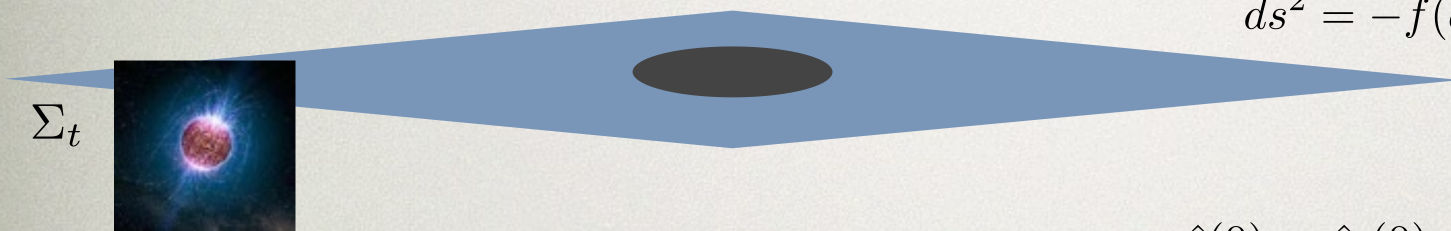
$\hat{\varrho}_\gamma(0) =$ thermal state with temperature T

$$\hat{\rho}(0) \underset{\hat{U}_{{\rm u}\gamma{\rm s}}(t)}{\longrightarrow} \hat{\rho}(t) \xrightarrow[\text{environment}]{\text{trace out the}} \hat{\rho}_{\rm u}(t) = {\rm tr}_{\gamma{\rm s}}\hat{\rho}(t)$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Initial state of the full system

$$\hat{\varrho}(0) = \hat{\varrho}_{\mathrm{s}}(0) \otimes \hat{\varrho}_{\mathrm{u}}(0) \otimes \hat{\varrho}_{\gamma}(0),$$

$$\hat{\varrho}_{\mathrm{s}}(0) = |0_{\mathrm{s}}\rangle\langle 0_{\mathrm{s}}| \qquad \hat{\varrho}_{\mathrm{u}}(0) = |0_{\mathrm{u}}\rangle\langle 0_{\mathrm{u}}|$$

$$\hat{\varrho}_{\gamma}(0) = \text{thermal state with temperature } T$$

$$\hat{\rho}(0) \underset{\hat{U}_{\mathrm{u}\gamma\mathrm{s}}(t)}{\longrightarrow} \hat{\rho}(t) \xrightarrow[\text{environment}]{\text{trace out the}} \hat{\rho}_{\mathrm{u}}(t) = \mathrm{tr}_{\gamma\mathrm{s}}\hat{\rho}(t)$$

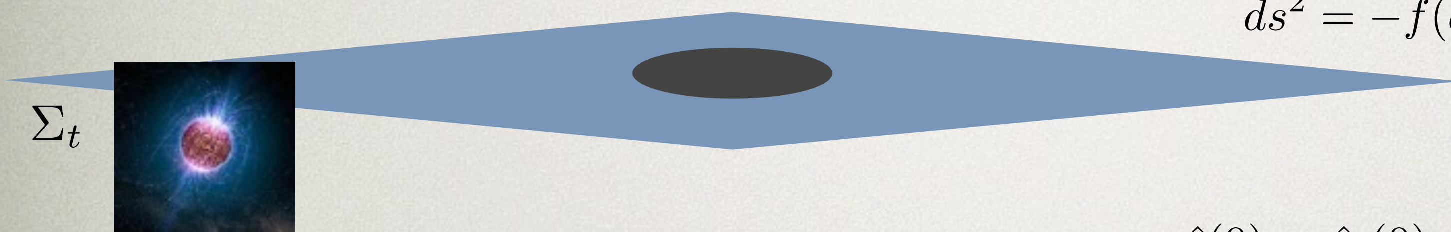
$$\varrho_{\mathrm{red}}(t, \varphi_{\mathrm{u}}, \varphi_{\mathrm{u}}') = \int d\psi_{\mathrm{u}} d\psi_{\mathrm{u}}' \varrho_{\mathrm{u}}(0, \psi_{\mathrm{u}}, \psi_{\mathrm{u}}') \times J_{\mathrm{red}}(t, \varphi_{\mathrm{u}}, \varphi_{\mathrm{u}}'; 0, \psi_{\mathrm{u}}, \psi_{\mathrm{u}}'),$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t



Initial state of the full system

$$\hat{\varrho}(0) = \hat{\varrho}_{\rm s}(0) \otimes \hat{\varrho}_{\rm u}(0) \otimes \hat{\varrho}_{\gamma}(0),$$

$$\hat{\varrho}_{\rm s}(0) = |0_{\rm s}\rangle\langle 0_{\rm s}| \qquad \hat{\varrho}_{\rm u}(0) = |0_{\rm u}\rangle\langle 0_{\rm u}|$$

$\hat{\varrho}_{\gamma}(0) =$ thermal state with temperature T

$$\hat{\rho}(0) \underset{\hat{U}_{{\rm u}\gamma{\rm s}}(t)}{\longrightarrow} \hat{\rho}(t) \xrightarrow[\text{environment}]{\text{trace out the}} \hat{\rho}_{\rm u}(t) = {\rm tr}_{\gamma{\rm s}}\hat{\rho}(t)$$

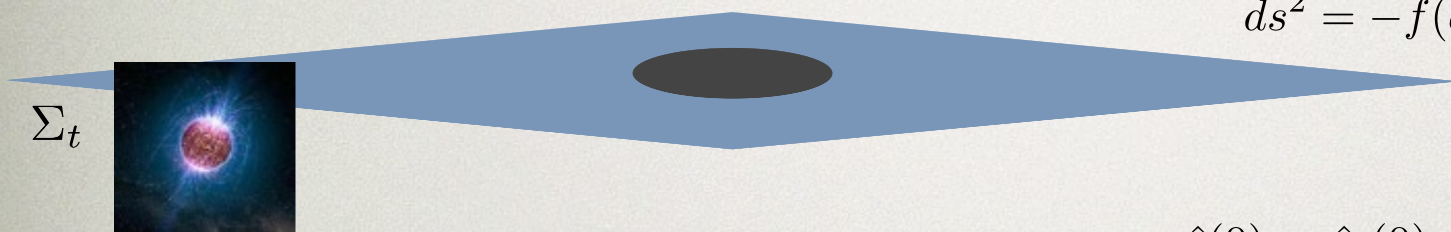
$$\varrho_{\rm u}(0, \psi_{\rm u}, {\psi_{\rm u}}') \equiv \langle \psi_{\rm u}|\hat{\varrho}_{\rm u}(0)|{\psi_{\rm u}}'\rangle$$

$$\varrho_{\rm red}(t, \varphi_{\rm u}, {\varphi_{\rm u}}') = \int d\psi_{\rm u} d{\psi_{\rm u}}' \varrho_{\rm u}(0, \psi_{\rm u}, {\psi_{\rm u}}') \quad \times J_{\rm red}(t, \varphi_{\rm u}, {\varphi_{\rm u}}'; 0, \psi_{\rm u}, {\psi_{\rm u}}'),$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Initial state of the full system

$$\hat{\varrho}(0) = \hat{\varrho}_{\rm s}(0) \otimes \hat{\varrho}_{\rm u}(0) \otimes \hat{\varrho}_\gamma(0),$$

$$\hat{\varrho}_{\rm s}(0) = |0_{\rm s}\rangle\langle 0_{\rm s}| \qquad \hat{\varrho}_{\rm u}(0) = |0_{\rm u}\rangle\langle 0_{\rm u}|$$

$\hat{\varrho}_\gamma(0) =$ thermal state with temperature T

$$\hat{\rho}(0) \underset{\hat{U}_{{\rm u}\gamma{\rm s}}(t)}{\longrightarrow} \hat{\rho}(t) \xrightarrow[\text{environment}]{\text{trace out the}} \hat{\rho}_{\rm u}(t) = {\rm tr}_{\gamma{\rm s}}\hat{\rho}(t)$$

$$\varrho_{\rm u}(0, \psi_{\rm u}, {\psi_{\rm u}}') \equiv \langle \psi_{\rm u}|\hat{\varrho}_{\rm u}(0)|{\psi_{\rm u}}'\rangle$$

$$\varrho_{\rm red}(t, \varphi_{\rm u}, {\varphi_{\rm u}}') = \int d\psi_{\rm u} d{\psi_{\rm u}}' \varrho_{\rm u}(0, \psi_{\rm u}, {\psi_{\rm u}}') \times J_{\rm red}(t, \varphi_{\rm u}, {\varphi_{\rm u}}'; 0, \psi_{\rm u}, {\psi_{\rm u}}'),$$

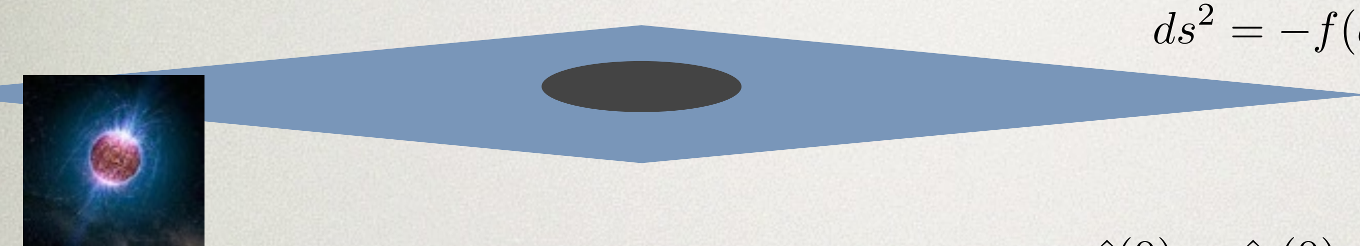
$$\hat{\phi}_{\rm u}(0, \mathbf{x})|\psi_{\rm u}\rangle = \psi_{\rm u}(\mathbf{x})|\psi_{\rm u}\rangle$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t



Initial state of the full system

$$\hat{\varrho}(0) = \hat{\varrho}_{\rm s}(0) \otimes \hat{\varrho}_{\rm u}(0) \otimes \hat{\varrho}_{\gamma}(0),$$

$$\hat{\varrho}_{\rm s}(0) = |0_{\rm s}\rangle\langle 0_{\rm s}| \qquad \hat{\varrho}_{\rm u}(0) = |0_{\rm u}\rangle\langle 0_{\rm u}|$$

$\hat{\varrho}_{\gamma}(0) = $ thermal state with temperature T

$$\hat{\rho}(0) \underset{\hat{U}_{{\rm u}\gamma{\rm s}}(t)}{\longrightarrow} \hat{\rho}(t) \xrightarrow{\text{trace out the environment}} \hat{\rho}_{\rm u}(t) = {\rm tr}_{\gamma{\rm s}}\hat{\rho}(t)$$

$$\varrho_{\rm u}(0,\psi_{\rm u},{\psi_{\rm u}}') \equiv \langle\psi_{\rm u}|\hat{\varrho}_{\rm u}(0)|{\psi_{\rm u}}'\rangle$$

$$\varrho_{\rm red}(t,\varphi_{\rm u},{\varphi_{\rm u}}') = \int d\psi_{\rm u} d{\psi_{\rm u}}' \varrho_{\rm u}(0,\psi_{\rm u},{\psi_{\rm u}}') \quad \times J_{\rm red}(t,\varphi_{\rm u},{\varphi_{\rm u}}';0,\psi_{\rm u},{\psi_{\rm u}}'),$$

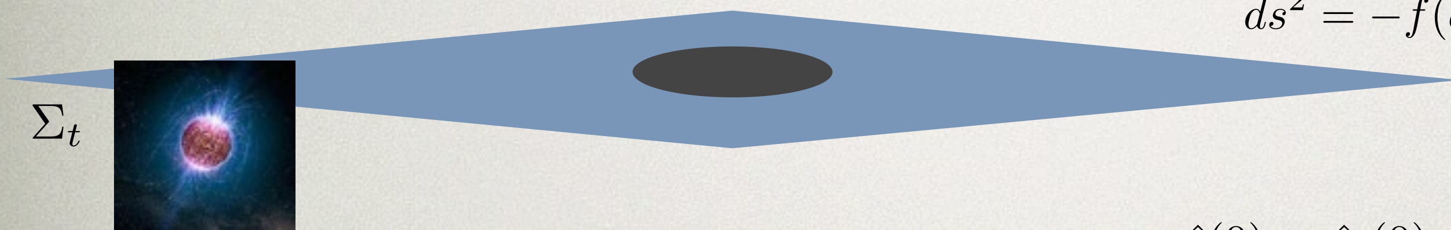
$$\hat{\phi}_{\rm u}(0,\mathbf{x})|\psi_{\rm u}\rangle = \psi_{\rm u}(\mathbf{x})|\psi_{\rm u}\rangle$$

$$J_{\rm red}(t,\varphi_{\rm u},{\varphi_{\rm u}}';0,\psi_{\rm u},{\psi_{\rm u}}') \equiv \int_{\psi_{\rm u}}^{\varphi_{\rm u}} \mathcal{D}\phi_{\rm u} \int_{{\psi_{\rm u}}'}^{{\varphi_{\rm u}}'} \mathcal{D}{\phi_{\rm u}}' e^{i\{S_\phi[\phi_{\rm u}] - S_\phi[{\phi_{\rm u}}']\}} F[\phi_{\rm u},{\phi_{\rm u}}']$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Initial state of the full system

$$\hat{\varrho}(0) = \hat{\varrho}_{\mathrm{s}}(0) \otimes \hat{\varrho}_{\mathrm{u}}(0) \otimes \hat{\varrho}_{\gamma}(0),$$

$$\hat{\varrho}_{\mathrm{s}}(0) = |0_{\mathrm{s}}\rangle\langle 0_{\mathrm{s}}| \qquad \hat{\varrho}_{\mathrm{u}}(0) = |0_{\mathrm{u}}\rangle\langle 0_{\mathrm{u}}|$$

$$\hat{\varrho}_{\gamma}(0) = \text{thermal state with temperature } T$$

$$\hat{\rho}(0) \underset{\hat{U}_{\mathrm{u}\gamma\mathrm{s}}(t)}{\longrightarrow} \hat{\rho}(t) \xrightarrow{\text{trace out the environment}} \hat{\rho}_{\mathrm{u}}(t) = \mathrm{tr}_{\gamma\mathrm{s}}\hat{\rho}(t)$$

Feynman-Vernon Influence Functional

$$\varrho_{\mathrm{red}}(t, \varphi_{\mathrm{u}}, \varphi_{\mathrm{u}}') = \int d\psi_{\mathrm{u}} d\psi_{\mathrm{u}}' \varrho_{\mathrm{u}}(0, \psi_{\mathrm{u}}, \psi_{\mathrm{u}}') \times J_{\mathrm{red}}(t, \varphi_{\mathrm{u}}, \varphi_{\mathrm{u}}'; 0, \psi_{\mathrm{u}}, \psi_{\mathrm{u}}'),$$

$$\hat{\phi}_{\mathrm{u}}(0, \mathbf{x})|\psi_{\mathrm{u}}\rangle = \psi_{\mathrm{u}}(\mathbf{x})|\psi_{\mathrm{u}}\rangle$$

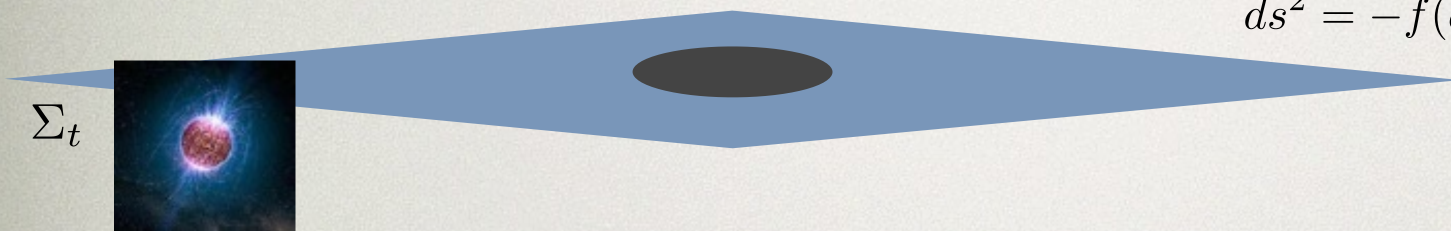
$$J_{\mathrm{red}}(t, \varphi_{\mathrm{u}}, \varphi_{\mathrm{u}}'; 0, \psi_{\mathrm{u}}, \psi_{\mathrm{u}}') \equiv \int_{\psi_{\mathrm{u}}}^{\varphi_{\mathrm{u}}} \mathcal{D}\phi_{\mathrm{u}} \int_{\psi_{\mathrm{u}}'}^{\varphi_{\mathrm{u}}'} \mathcal{D}\phi_{\mathrm{u}}' e^{i\{S_{\phi}[\phi_{\mathrm{u}}] - S_{\phi}[\phi_{\mathrm{u}}']\}} F[\phi_{\mathrm{u}}, \phi_{\mathrm{u}}']$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

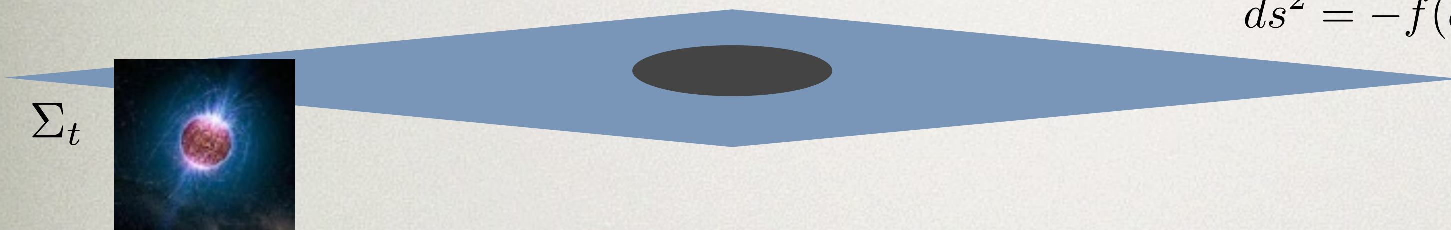
Σ_t



From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Σ_t

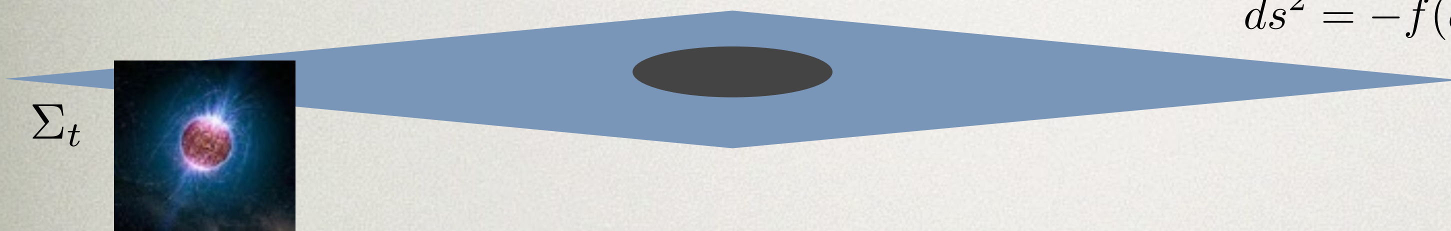
$$F[\phi_{\rm u}, {\phi_{\rm u}}'] \equiv \int d\varphi_{\rm s} \int d\psi_{\rm s} d{\psi_{\rm s}}' \varrho_{\rm s}(0, \psi_{\rm s}, {\psi_{\rm s}}') \int_{\psi_{\rm s}, {\psi_{\rm s}}'}^{\varphi_{\rm s} = {\varphi_{\rm s}}'} \mathcal{D}\phi_{\rm s} \mathcal{D}{\phi_{\rm s}}' e^{i\{S_\phi[\phi_{\rm s}] - S_\phi[{\phi_{\rm s}}']\}} \tilde{F}[\phi_{\rm s} + \phi_{\rm u}, {\phi_{\rm s}}' + {\phi_{\rm u}}'],$$

$$\tilde{F}[\phi, \phi'] \equiv \int d\varsigma_{ab} \int d\xi_{ab} d{\xi_{ab}}' \varrho_\gamma(0, \xi_{ab}, {\xi_{ab}}') \int_{\xi_{ab}, {\xi_{ab}}'}^{\varsigma_{ab} = {\varsigma_{ab}}'} \mathcal{D}\gamma_{ab} \mathcal{D}{\gamma_{ab}}' e^{i\{S_\gamma[\gamma_{ab}] - S_\gamma[{\gamma_{ab}}']\}} e^{i\{S_{\rm int}[\phi, \gamma_{ab}] - S_{\rm int}[\phi', {\gamma_{ab}}']\}}$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

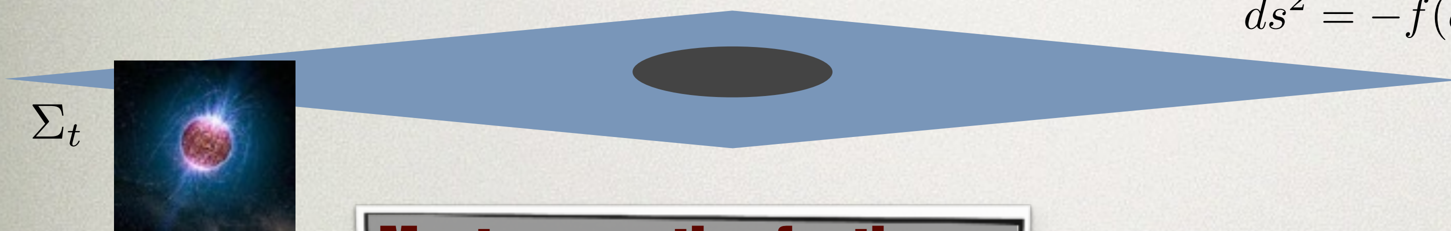


From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t

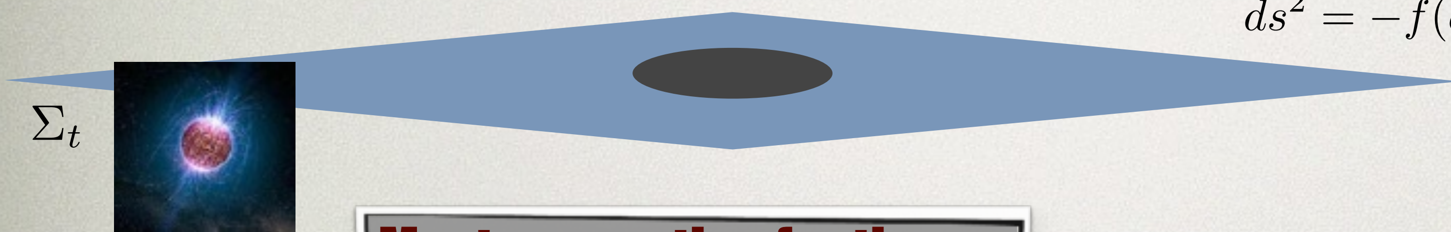


Master equation for the state of the unstable mode

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Master equation for the state of the unstable mode

$$\partial_t \varrho_{\rm red} = -i\left[\frac{1}{2}(-\partial_q^2 + \partial_{q'}^2) - \frac{\Omega^2}{2}(q^2 - q'^2)\right]\varrho_{\rm red} - \frac{\kappa^2}{4}\int_0^t d\tau[\mathrm{Re}\langle\hat{Q}(\tau)\hat{Q}(0)\rangle\cosh\Omega\tau - \Omega\mathrm{Re}\langle\hat{Q}(\tau)\hat{P}(0)\rangle\sinh\Omega\tau](q-q')^2\varrho_{\rm red}$$

$$- \frac{\kappa^2}{4}\int_0^t d\tau[\mathrm{Re}\langle\hat{P}(\tau)\hat{P}(0)\rangle\cosh\Omega\tau - \Omega^{-1}\mathrm{Re}\langle\hat{P}(\tau)\hat{Q}(0)\rangle\sinh\Omega\tau](-i\partial_q - i\partial_{q'})^2\varrho_{\rm red}$$

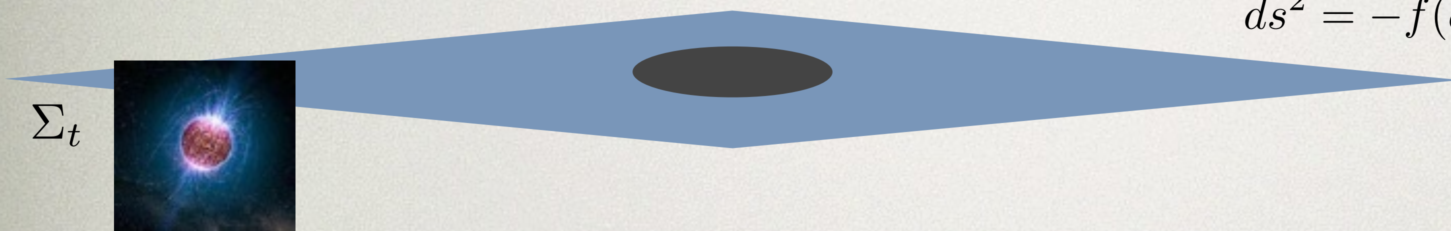
$$+ \frac{\kappa^2}{4}\int_0^t d\tau\{[\Omega^{-1}\mathrm{Re}\langle\hat{Q}(\tau)\hat{Q}(0)\rangle + \Omega\mathrm{Re}\langle\hat{P}(\tau)\hat{P}(0)\rangle]\sinh\Omega\tau$$

$$- [\mathrm{Re}\langle\hat{P}(\tau)\hat{Q}(0)\rangle + \mathrm{Re}\langle\hat{Q}(\tau)\hat{P}(0)\rangle]\cosh\Omega\tau\}(q-q')(-i\partial_q - i\partial_{q'})\varrho_{\rm red} + \dots$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



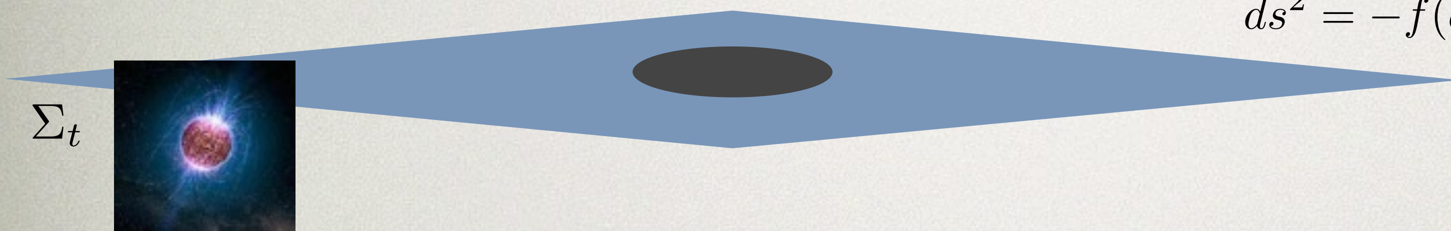
Σ_t

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t

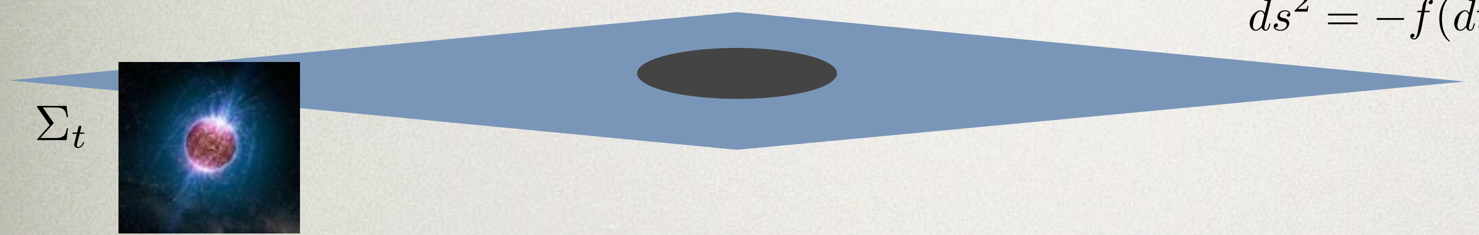


Wigner function for the unstable mode

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Wigner function for the unstable mode

$$\partial_t W_{\rm red} = \{H(q,p), W_{\rm red}\} + \frac{\kappa^2}{4}\int_0^t d\tau [\mathrm{Re}\langle \hat{Q}(\tau)\hat{Q}(0)\rangle \cosh\Omega\tau - \Omega \mathrm{Re}\langle \hat{Q}(\tau)\hat{P}(0)\rangle \sinh\Omega\tau]\partial_p^2 W_{\rm red}$$

$$+\frac{\kappa^2}{4}\int_0^t d\tau [\mathrm{Re}\langle \hat{P}(\tau)\hat{P}(0)\rangle \cosh\Omega\tau - \Omega^{-1} \mathrm{Re}\langle \hat{P}(\tau)\hat{Q}(0)\rangle \sinh\Omega\tau]\partial_q^2 W_{\rm red}$$

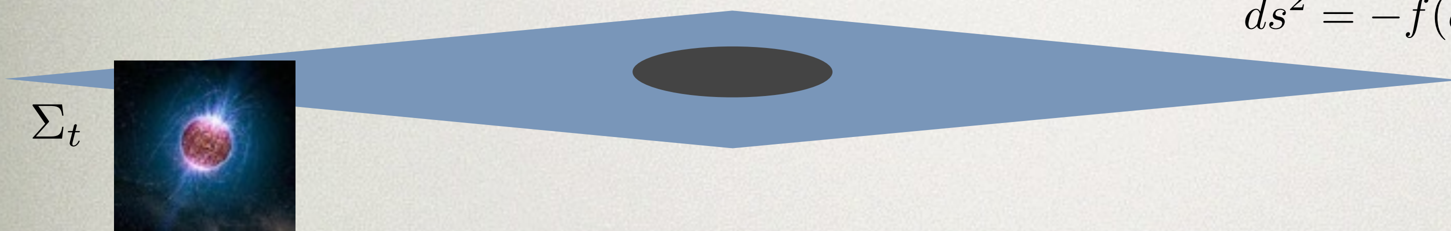
$$+\frac{\kappa^2}{4}\int_0^t d\tau \{[\Omega^{-1}\mathrm{Re}\langle \hat{Q}(\tau)\hat{Q}(0)\rangle + \Omega \mathrm{Re}\langle \hat{P}(\tau)\hat{P}(0)\rangle] \sinh\Omega\tau$$

$$-[\mathrm{Re}\langle \hat{P}(\tau)\hat{Q}(0)\rangle + \mathrm{Re}\langle \hat{Q}(\tau)\hat{P}(0)\rangle] \cosh\Omega\tau\}\partial_p\partial_q W_{\rm red} + \dots$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

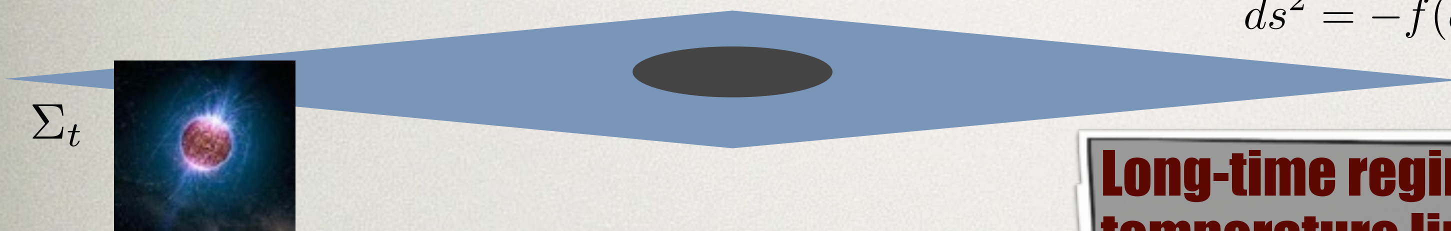


Σ_t

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



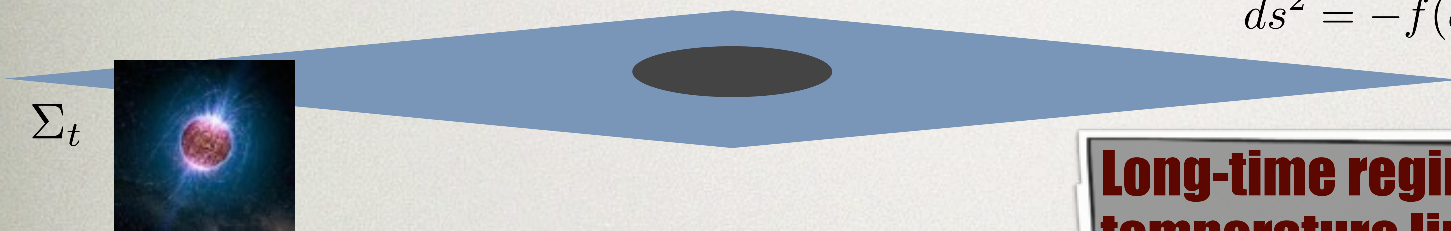
Long-time regime and high temperature limit

$\Omega t \gg 1$ $k_{\mathrm{B}} T \gg \Omega$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Long-time regime and high temperature limit

$\Omega t \gg 1$ $\quad k_{\mathrm{B}} T \gg \Omega$

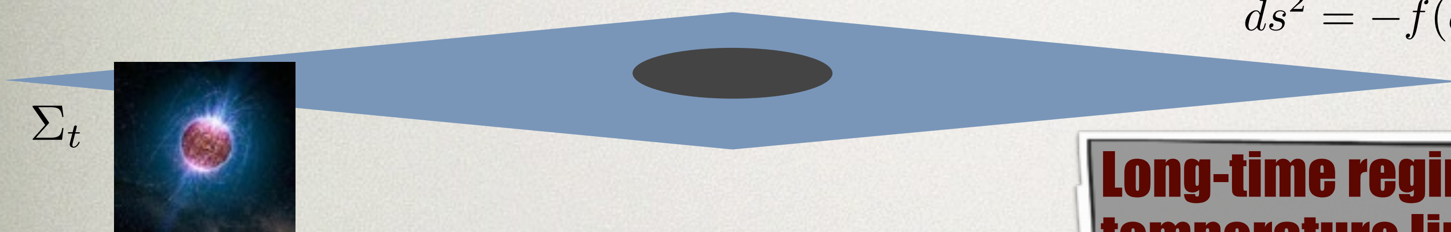
Master equation for the state of the unstable mode

$$\partial_t \varrho_{\mathrm{red}} \approx -i\left[\frac{1}{2}(-\partial_q^2 + \partial_{q'}^2) - \frac{\Omega^2}{2}(q^2 - q'^2)\right]\varrho_{\mathrm{red}} - D(q-q')^2\varrho_{\mathrm{red}} + \ldots$$

From quantum to classical

* Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Long-time regime and high temperature limit

$\Omega t \gg 1 \qquad k_{\mathrm{B}} T \gg \Omega$

Master equation for the state of the unstable mode

$$\partial_t \varrho_{\mathrm{red}} \approx -i\left[\frac{1}{2}(-\partial_q^2 + \partial_{q'}^2) - \frac{\Omega^2}{2}(q^2 - q'^2)\right]\varrho_{\mathrm{red}} - D(q-q')^2\varrho_{\mathrm{red}} + \dots$$

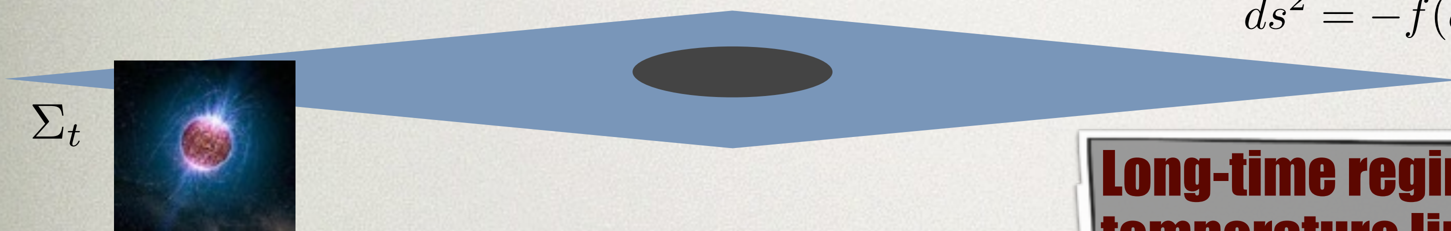
Wigner function for the unstable mode

$$\partial_t W_{\mathrm{red}} \approx \{H(q,p), W_{\mathrm{red}}\} + D\partial_p^2 W_{\mathrm{red}} + \dots$$

FROM QUANTUM TO CLASSICAL

* Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Σ_t

Long-time regime and high temperature limit

$$\Omega t \gg 1 \qquad k_{\mathrm{B}} T \gg \Omega$$

Master equation for the state of the unstable mode

$$\partial_t \varrho_{\mathrm{red}} \approx -i\left[\frac{1}{2}(-\partial_q^2 + \partial_{q'}^2) - \frac{\Omega^2}{2}(q^2 - q'^2)\right]\varrho_{\mathrm{red}} - D(q-q')^2\varrho_{\mathrm{red}} + \ldots$$

$$D = 8\pi\Delta\left(\frac{\ell_{\mathrm{P}}}{\mathrm{R}}\right)^2\left(\frac{k_{\mathrm{B}}T}{\Omega}\right)\Omega^2$$

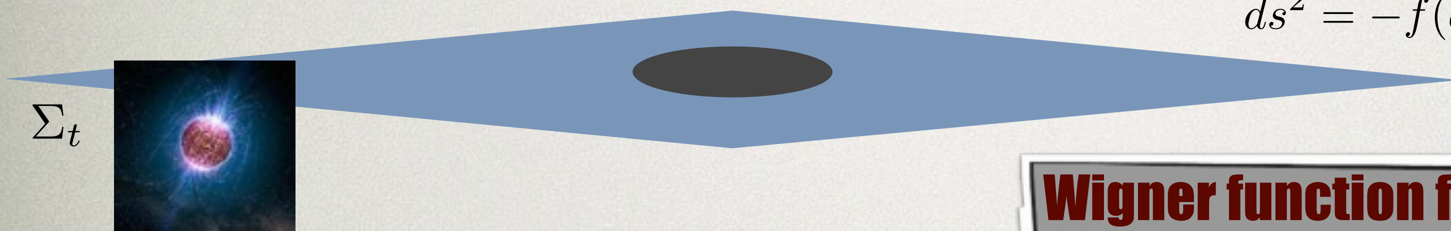
Wigner function for the unstable mode

$$\partial_t W_{\mathrm{red}} \approx \{H(q,p), W_{\mathrm{red}}\} + D\partial_p^2 W_{\mathrm{red}} + \ldots$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Wigner function for the unstable mode

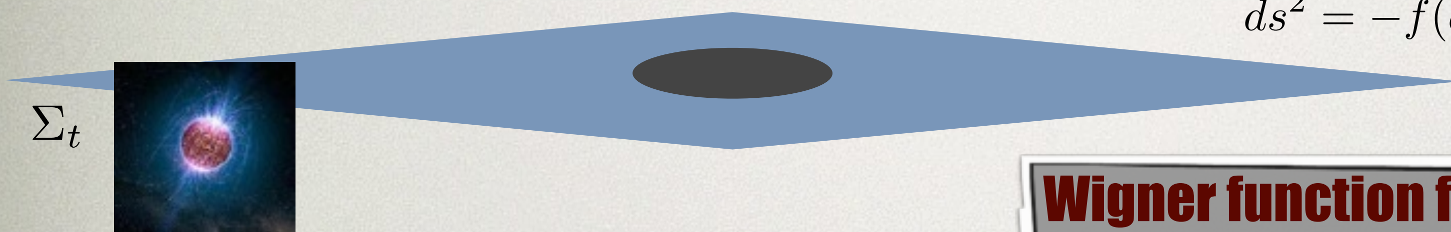
$$\partial_t W_{\rm red} \approx \{H(q,p), W_{\rm red}\} + D\partial_p^2 W_{\rm red} + \dots$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t



Wigner function for the unstable mode

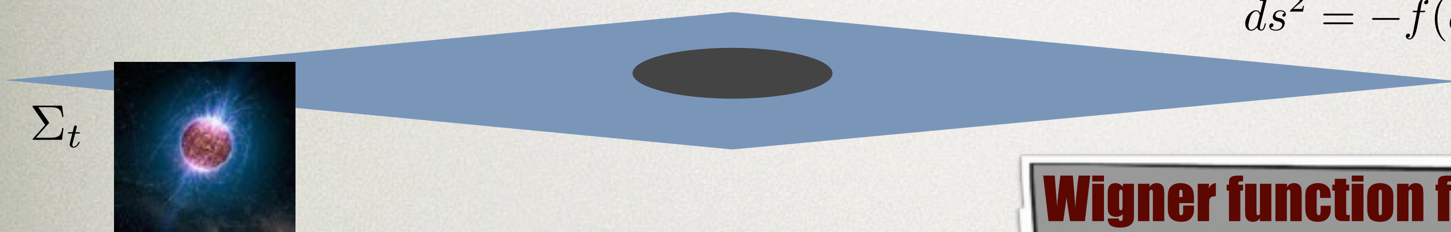
$$\partial_t W_{\rm red} \approx \{H(q,p), W_{\rm red}\} + D\partial_p^2 W_{\rm red} + \ldots$$

- L. Diósi and C. Kiefer J.Phys.A (2002)
- O. Brodier and A. Ozorio de Almeida, PRE (2004)

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Wigner function for the unstable mode

- L. Diósi and C. Kiefer J.Phys.A (2002)

$$\partial_t W_{\rm red} \approx \{H(q,p), W_{\rm red}\} + D\partial_p^2 W_{\rm red} + \ldots$$

- O. Brodier and A. Ozorio de Almeida, PRE (2004)

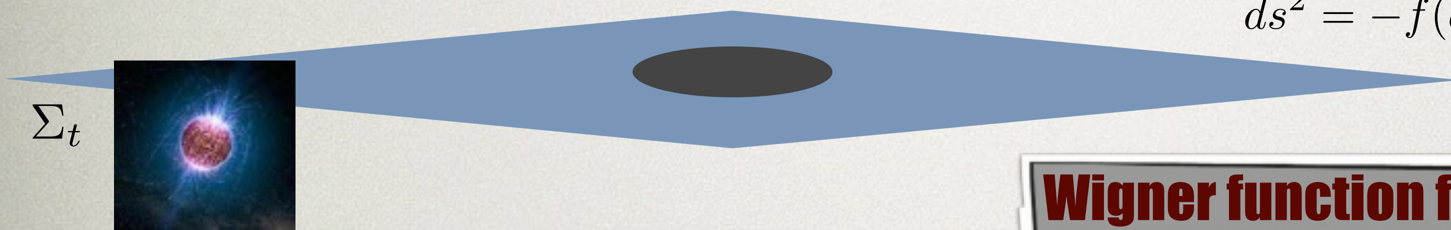
$$W_{\rm red}(t > t_{\rm d}) > 0 \text{ (regardless of the initial state)}$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t



Wigner function for the unstable mode

- L. Diósi and C. Kiefer J.Phys.A (2002)

$$\partial_t W_{\rm red} \approx \{H(q,p), W_{\rm red}\} + D\partial_p^2 W_{\rm red} + \ldots$$

- O. Brodier and A. Ozorio de Almeida, PRE (2004)

$$W_{\rm red}(t > t_{\rm d}) > 0 \text{ (regardless of the initial state)}$$

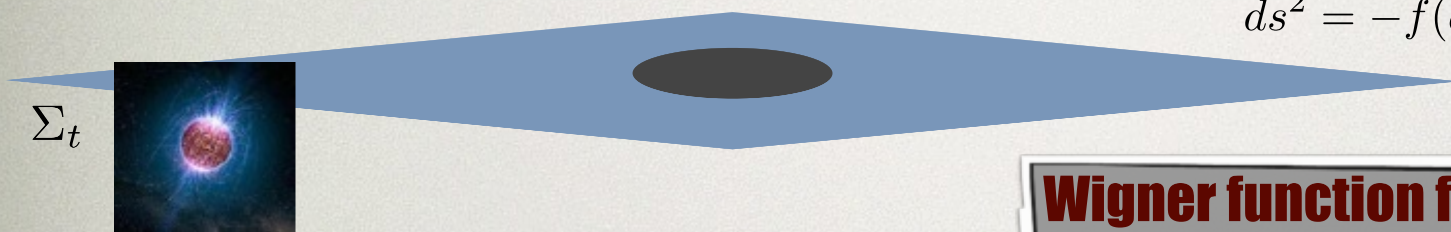
$$t_{\rm d} \sim \frac{1}{\Omega} \ln\left[\frac{1}{8\pi\Delta}\left(\frac{\mathrm{R}}{\ell_{\rm P}}\right)^2 \frac{\Omega}{k_{\rm B}T}\right]$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t



Wigner function for the unstable mode

- L. Diósi and C. Kiefer J.Phys.A (2002)

$$\partial_t W_{\rm red} \approx \{H(q,p), W_{\rm red}\} + D\partial_p^2 W_{\rm red} + \ldots$$

- O. Brodier and A. Ozorio de Almeida, PRE (2004)

$$W_{\rm red}(t > t_{\rm d}) > 0 \text{ (regardless of the initial state)}$$

$$t_{\rm d} \sim \frac{1}{\Omega} \ln\left[\frac{1}{8\pi\Delta}\left(\frac{\mathrm{R}}{\ell_{\rm P}}\right)^2 \frac{\Omega}{k_{\rm B}T}\right]$$

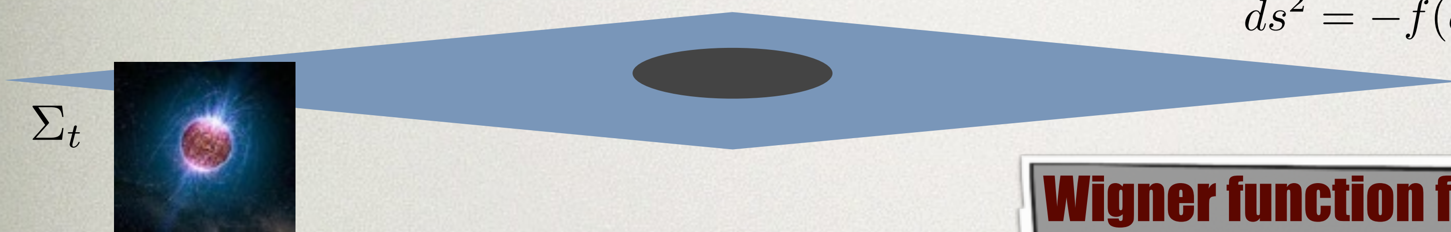
- For a R~10km neutron star and T~1K

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t



Wigner function for the unstable mode

- L. Diósi and C. Kiefer J.Phys.A (2002)

$$\partial_t W_{\rm red} \approx \{H(q,p), W_{\rm red}\} + D\partial_p^2 W_{\rm red} + \dots$$

- O. Brodier and A. Ozorio de Almeida, PRE (2004)

$$W_{\rm red}(t > t_{\rm d}) > 0 \text{ (regardless of the initial state)}$$

$$t_{\rm d} \sim \frac{1}{\Omega}\ln\left[\frac{1}{8\pi\Delta}\left(\frac{\mathrm{R}}{\ell_{\rm P}}\right)^2\frac{\Omega}{k_{\rm B}T}\right]$$

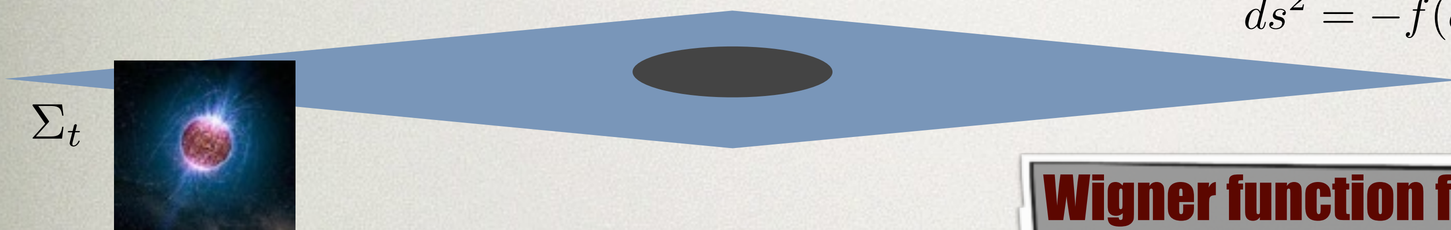
- For a R~10km neutron star and T~1K

$$t_{\rm d} \sim 160 \times \Omega^{-1} \sim 160 \times \mathrm{R}$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Σ_t

Wigner function for the unstable mode

$$\partial_t W_{\rm red} \approx \{H(q,p), W_{\rm red}\} + D\partial_p^2 W_{\rm red} + \dots$$

- L. Diósi and C. Kiefer J.Phys.A (2002)
- O. Brodier and A. Ozorio de Almeida, PRE (2004)

$$W_{\rm red}(t > t_{\rm d}) > 0 \text{ (regardless of the initial state)}$$

$$t_{\rm d} \sim \frac{1}{\Omega} \ln\left[\frac{1}{8\pi\Delta}\left(\frac{\mathrm{R}}{\ell_{\rm P}}\right)^2 \frac{\Omega}{k_{\rm B}T}\right]$$

- For a R~10km neutron star and T~1K

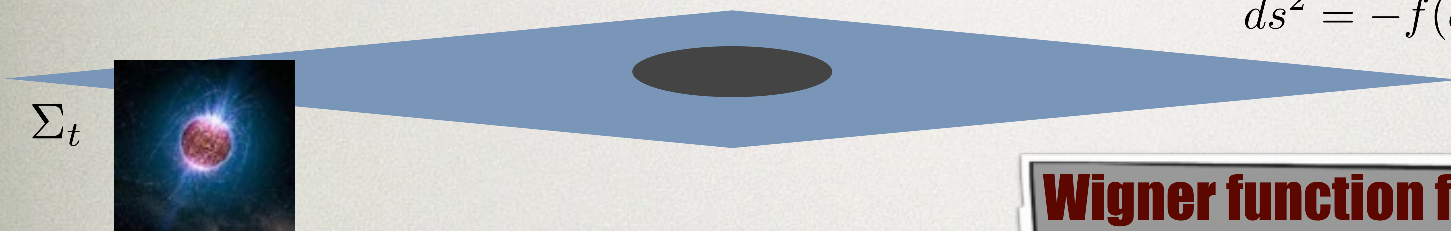
$t_{\rm d} \sim 160 \times \Omega^{-1} \sim 160 \times \mathrm{R}$ which is of the order of

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t



Wigner function for the unstable mode

- L. Diósi and C. Kiefer J.Phys.A (2002)

$$\partial_t W_{\rm red} \approx \{H(q,p), W_{\rm red}\} + D\partial_p^2 W_{\rm red} + \ldots$$

- O. Brodier and A. Ozorio de Almeida, PRE (2004)

$$W_{\rm red}(t > t_{\rm d}) > 0 \text{ (regardless of the initial state)}$$

$$t_{\rm d} \sim \frac{1}{\Omega} \ln\left[\frac{1}{8\pi\Delta}\left(\frac{\mathrm{R}}{\ell_{\rm P}}\right)^2 \frac{\Omega}{k_{\rm B}T}\right]$$

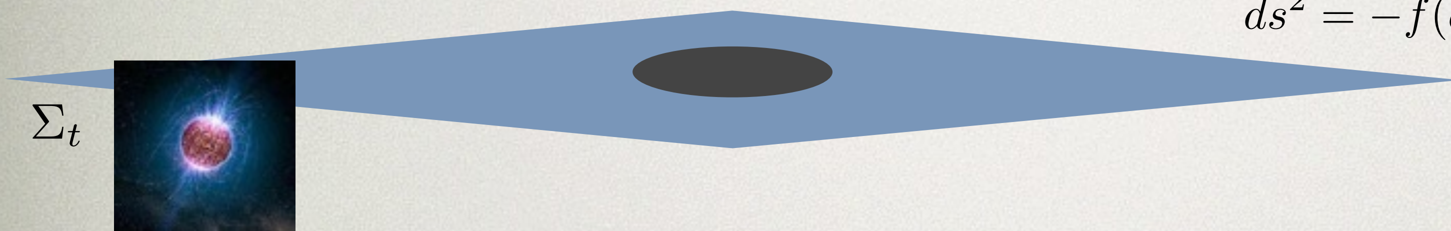
- For a R~10km neutron star and T~1K

$t_{\rm d} \sim 160 \times \Omega^{-1} \sim 160 \times \mathrm{R}$ which is of the order of $t_{\rm br} \sim 10^{-3}\,\mathrm{s}$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$



Σ_t

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t

Master equation for the state of the unstable mode

$$\partial_t \varrho_{\rm red} \approx -i\left[\frac{1}{2}(-\partial_q^2 + \partial_{q'}^2) - \frac{\Omega^2}{2}(q^2 - q'^2)\right]\varrho_{\rm red} - D(q-q')^2\varrho_{\rm red} + \dots$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t

Master equation for the state of the unstable mode

$$\partial_t \varrho_{\rm red} \approx -i\left[\frac{1}{2}(-\partial_q^2 + \partial_{q'}^2) - \frac{\Omega^2}{2}(q^2 - q'^2)\right]\varrho_{\rm red} - D(q-q')^2\varrho_{\rm red} + \dots$$

$$\varrho_{\rm red}(t,q,q') \approx 2e^{-2\sigma^2(q-q')^2 - i\Omega(q^2-q'^2)/2}e^{-\Omega t}\int_{-\infty}^{+\infty} du'\tilde{W}_{\rm red}(0,u',0)$$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t

Master equation for the state of the unstable mode

$$\partial_t \varrho_{\rm red} \approx -i\left[\frac{1}{2}(-\partial_q^2 + \partial_{q'}^2) - \frac{\Omega^2}{2}(q^2 - q'^2)\right]\varrho_{\rm red} - D(q-q')^2\varrho_{\rm red} + \dots$$

$$\varrho_{\rm red}(t,q,q') \approx 2e^{-2\sigma^2(q-q')^2 - i\Omega(q^2-q'^2)/2}e^{-\Omega t}\int_{-\infty}^{+\infty} du'\tilde{W}_{\rm red}(0,u',0)$$

mixture of localized states with width: $(2\sigma)^{-1} \sim \frac{\ell_{\rm P}}{\rm R}\sqrt{k_{\rm B}T}$

From quantum to classical

* Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t

Master equation for the state of the unstable mode

$$\partial_t \varrho_{\rm red} \approx -i\left[\frac{1}{2}(-\partial_q^2 + \partial_{q'}^2) - \frac{\Omega^2}{2}(q^2 - q'^2)\right]\varrho_{\rm red} - D(q-q')^2\varrho_{\rm red} + \dots$$

$$\varrho_{\rm red}(t,q,q') \approx 2e^{-2\sigma^2(q-q')^2 - i\Omega(q^2-q'^2)/2}e^{-\Omega t}\int_{-\infty}^{+\infty} du'\tilde{W}_{\rm red}(0,u',0)$$

mixture of localized states with width: $(2\sigma)^{-1} \sim \frac{\ell_{\rm P}}{\rm R}\sqrt{k_{\rm B}T}$

* for each such states

$\rho_{\rm qf} \sim \sigma^{-2}\Omega^2/{\rm R}^3$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t

Master equation for the state of the unstable mode

$$\partial_t \varrho_{\rm red} \approx -i\left[\frac{1}{2}(-\partial_q^2 + \partial_{q'}^2) - \frac{\Omega^2}{2}(q^2 - q'^2)\right]\varrho_{\rm red} - D(q-q')^2\varrho_{\rm red} + \ldots$$

$$\varrho_{\rm red}(t,q,q') \approx 2e^{-2\sigma^2(q-q')^2 - i\Omega(q^2-q'^2)/2}e^{-\Omega t}\int_{-\infty}^{+\infty} du'\tilde{W}_{\rm red}(0,u',0)$$

mixture of localized states with width: $(2\sigma)^{-1} \sim \frac{\ell_{\rm P}}{\mathrm{R}}\sqrt{k_{\rm B}T}$

- for each such states

$\rho_{\rm qf} \sim \sigma^{-2}\Omega^2/\mathrm{R}^3$ $\qquad$ $\frac{\rho_{\rm qf}}{\rho} \sim \frac{\Omega}{k_{\rm B}T} \sim 10^{-7}$

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Σ_t

Master equation for the state of the unstable mode

$$\partial_t \varrho_{\rm red} \approx -i\left[\frac{1}{2}(-\partial_q^2 + \partial_{q'}^2) - \frac{\Omega^2}{2}(q^2 - q'^2)\right]\varrho_{\rm red} - D(q-q')^2\varrho_{\rm red} + \ldots$$

$$\varrho_{\rm red}(t,q,q') \approx 2e^{-2\sigma^2(q-q')^2 - i\Omega(q^2-q'^2)/2}e^{-\Omega t}\int_{-\infty}^{+\infty} du'\tilde{W}_{\rm red}(0,u',0)$$

mixture of localized states with width: $(2\sigma)^{-1} \sim \frac{\ell_{\rm P}}{\mathrm{R}}\sqrt{k_{\rm B}T}$

- for each such states

$\rho_{\rm qf} \sim \sigma^{-2}\Omega^2/\mathrm{R}^3 \qquad \frac{\rho_{\rm qf}}{\rho} \sim \frac{\Omega}{k_{\rm B}T} \sim 10^{-7}$

Pointer states are narrow enough (in the long-time regime) to be approximated by classical states

From quantum to classical

- Unstable Phase

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2\theta \, d\varphi^2)$$

Σ_t

Master equation for the state of the unstable mode

$$\partial_t \varrho_{\rm red} \approx -i \left[\frac{1}{2}(\ldots$$

$$\varrho_{\rm red}(t, q, q') \approx \ldots$$

THUS, BY THE TIME BACKREACTION BECOMES INELUCTABLE THE (UNSTABLE SECTOR OF THE) INITIALLY PURE VACUUM STATE HAS EVOLVED INTO A STATISTICAL MIXTURE OF LOCALIZED STATES IN THE FIELD AMPLITUDE AND MOMENTUM REPRESENTATION.

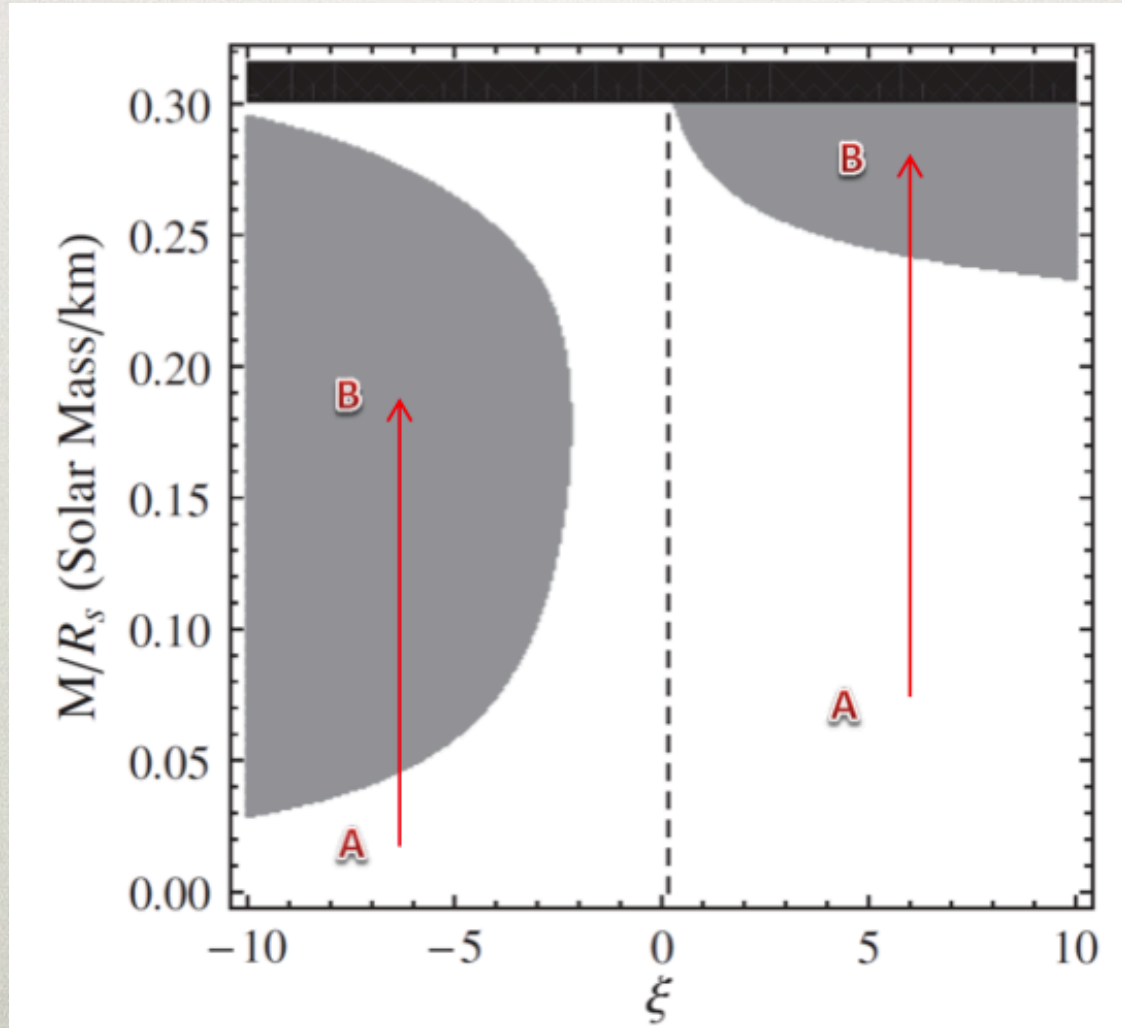
mixture of localized states with width: $(2\sigma)^{-1} \sim \frac{\ell_{\rm P}}{\rm R}\sqrt{k_{\rm B}T}$

- for each such states

$$\rho_{\rm qf} \sim \sigma^{-2}\Omega^2/{\rm R}^3 \qquad \frac{\rho_{\rm qf}}{\rho} \sim \frac{\Omega}{k_{\rm B}T} \sim 10^{-7}$$

Pointer states are narrow enough (in the long-time regime) to be approximated by classical states

Backreaction



Backreaction

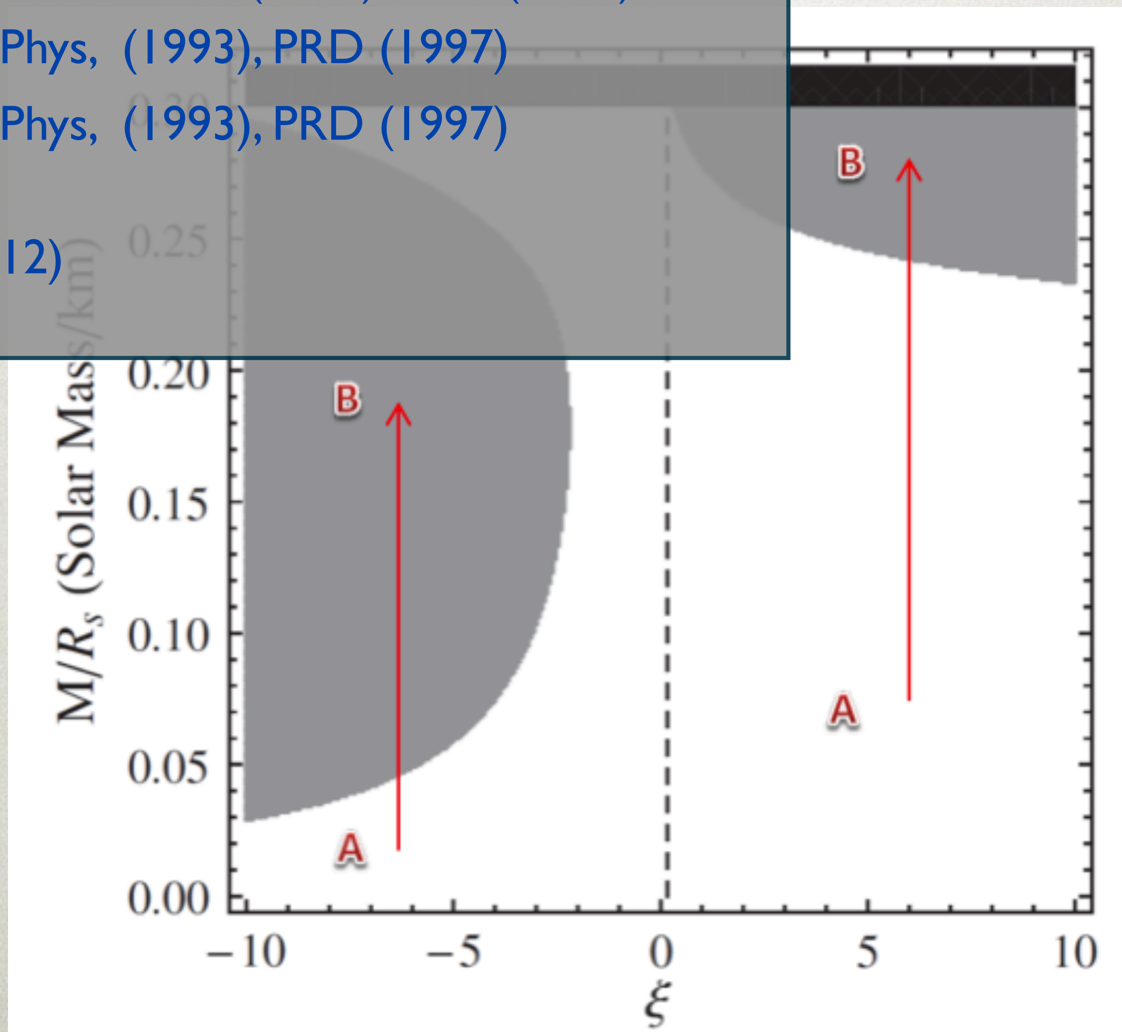
T. Damour, E. Esposito-Farèse, PRL (1993), PRD (1996)

T. Harada, Prog. Theor. Phys, (1993), PRD (1997)

T. Harada, Prog. Theor. Phys, (1993), PRD (1997)

Pani et. al., PRD (2011)

M. Ruiz et. al., PRD (2012)



Backreaction

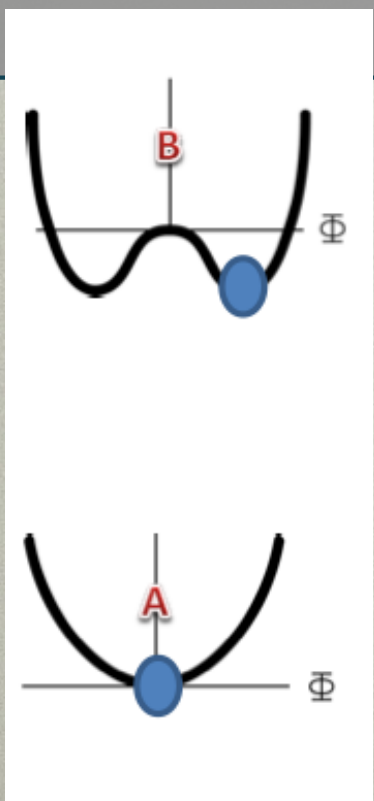
T. Damour, E. Esposito-Farèse, PRL (1993), PRD (1996)

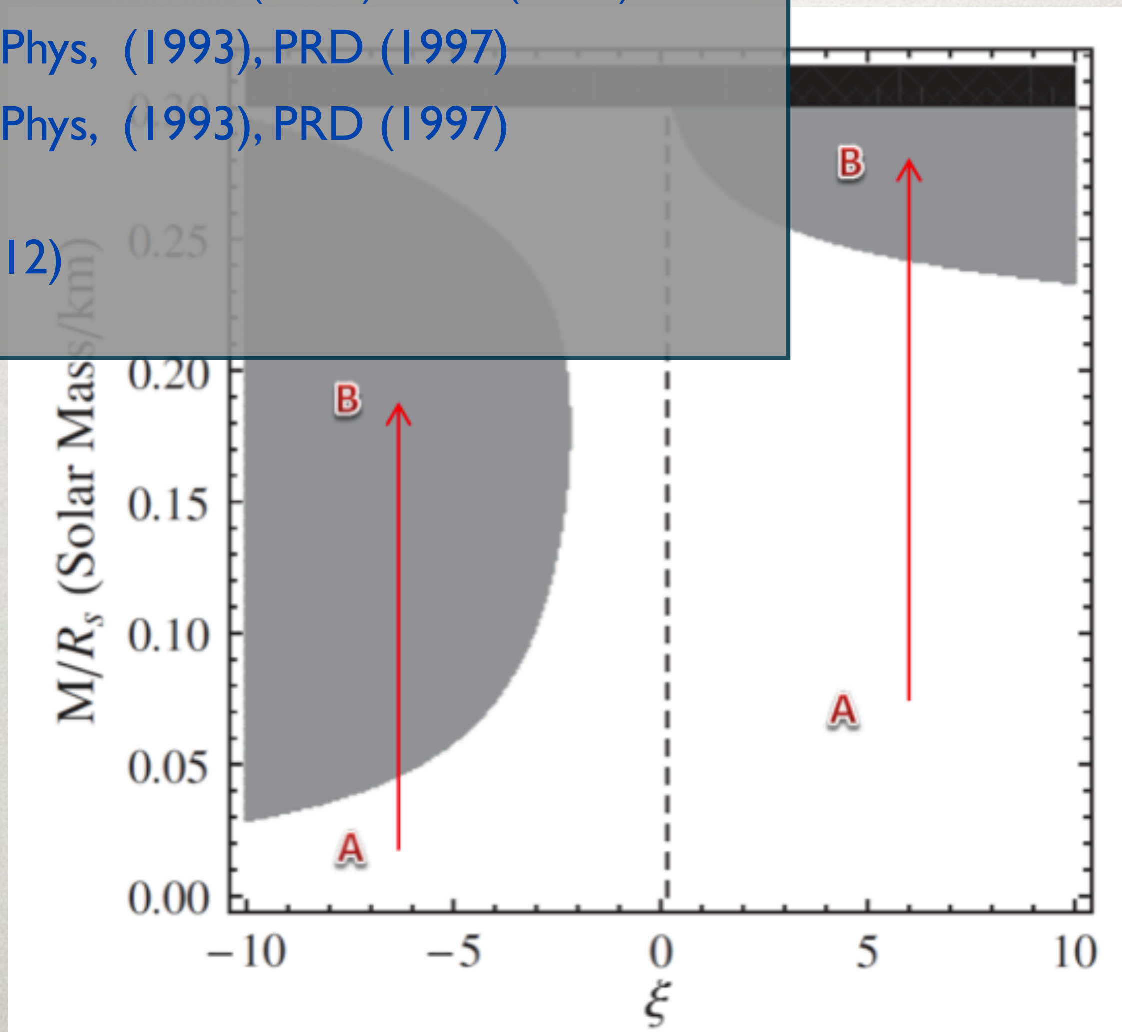
T. Harada, Prog. Theor. Phys, (1993), PRD (1997)

T. Harada, Prog. Theor. Phys, (1993), PRD (1997)

Pani et. al., PRD (2011)

M. Ruiz et. al., PRD (2012)





Backreaction

T. Damour, E. Esposito-Farèse, PRL (1993), PRD (1996)

T. Harada, Prog. Theor. Phys, (1993), PRD (1997)

T. Harada, Prog. Theor. Phys, (1993), PRD (1997)

Pani et. al., PRD (2011)

M. Ruiz et. al., PRD (2012)