Determination of the expansion rate from Type Ia Supernovae: a model-independent approach

Sandra Benitez-Herrera

W. Hillebrandt, E. Ishida, M. Maturi

Challenges Workshop – Instituto de Física, UFRJ

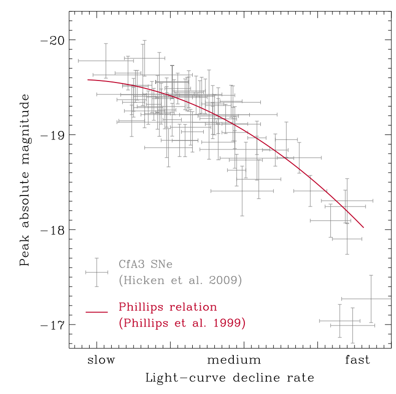
May 26, 2015

Type Ia Supernovae

- Standardizable candles. Can be calibrated through the width-luminosity relation *(Phillips 1993)*
- Brighter events decline slower, fainter decay faster
- Distance modulus

$$\mu_B = m_B + \alpha x_1 - \beta c - M_B$$

Luminosity Distance $D_{\mathrm{L}}(z) = 10^{\mu/5+1}$



(S. Taubenberger)

"Model-independent"?

- **Usual Approach**: Cosmological parameters are obtained from fits to observations and model predictions are validated

$$H^2(a) \equiv H_0^2 E^2(a) = H_0^2 \left[\frac{\Omega_{\mathrm{r}0}}{a^4} + \frac{\Omega_{\mathrm{m}0}}{a^3} - \frac{\Omega_{k0}}{a^2} + \Omega_{\Lambda 0} \right]$$

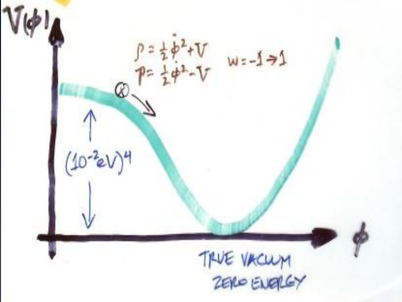
- SNe Ia directly test geometry. No need to define a Friedman Equation a priori
- Direct reconstruction of D_{L} and H
- Literature: *Starobinsky 1998; Huterer & Turner 1999, 2001; Shafieloo 2006, 2007, Ishida et al. 2011...*

Motivation

- Geometrical framework to compare different cosmological theories

- vacuum energy: $w = -1$
- dynamically evolving scalar-field: $w(a) = w_0 + w_a(1 - a)$
- Modified Gravity: terms on the left side of Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$



The Method

From Robertson-Walker metric $[a = (1+z)^{-1}$ and $e(a) = 1/E(a)]$

$$D_{\mathrm{L}}(a) = \frac{c}{H_0}\frac{1}{a}\int_a^1 \frac{e(x)}{x^2}dx \xrightarrow[d/da]{} D'_{\mathrm{L}}(a) = -\frac{1}{a^2}\int_a^1 \frac{e(x)}{x^2}dx - \frac{e(a)}{a^3}$$

Volterra Integral Equation

$$e(a) = f(a) + \lambda \int_1^x K(x,t)e(t)dt$$

$$e(a) = -a^3 D'_{\mathrm{L}} + a \int_1^a x^{-2} e(x)dx$$

Neumann Series: (*Arfken & Weber 1995*)

$$e(a) = e_0 + \sum_{n=1}^{\infty} \lambda^n e_n(a)$$

$$e_0 = -a^3 D'_{\mathrm{L}}(a)$$

$$e_n = \int_1^a x^{-2} e_{n-1}(t)dt$$

The Method

Problem: equation involves $D'_{\rm L}$!

- First: smooth the data

$$D_{\rm L}(a) = \sum_{j=0}^{J} c_j p_j(a) \quad \rightarrow \quad D'_{\rm L}(a) = \sum_{j=0}^{J} c_j \, p'_j(a)$$

- Approximate this derivative to the derivative of the real data
- Principal Component Analysis to derive an *optimal* basis
- The expansion coefficients are found minimizing

$$\chi^2 = \sum_{i}^{\rm SNe} \frac{[D_{\rm L}|_{{\rm obs},i} - D_{\rm L}(a_i, c_j, H_0)]^2}{\sigma_i^2}$$

PC Approach

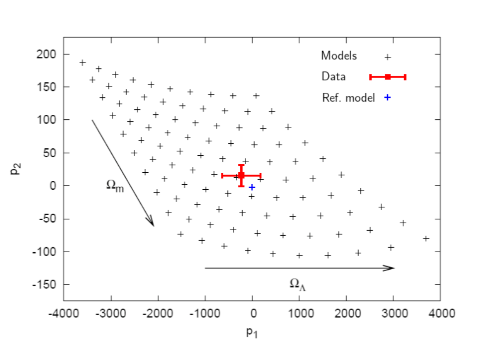
1. Construct **Training set** t_i that samples all possible behaviours of the observable
2. Define a **Reference vector** $\bar{t}$ which will be the origin of the new parameter space
3. Construct **Scatter Matrix** (analogous to a covariance matrix): $S = \Delta\Delta^T$ with $\Delta = (t_1 - \bar{t}, t_2 - \bar{t}, ..., t_M - \bar{t})$ containing the differences between each training vector t_i and $\bar{t}$
4. Principal Components, w_i, calculated by solving the eigenvalue problem: $w_i = \lambda_i S w_i$
5. Optimal basis for that training set $\rightarrow$ concentrates information about deviations from the reference model in a few features (by definition)

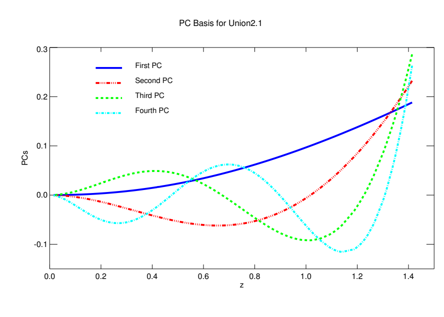
The PC Basis

Initial Matrix:

$0.1 < \Omega_m < 0.5$; $0.5 < \Omega_\Lambda < 0.9$

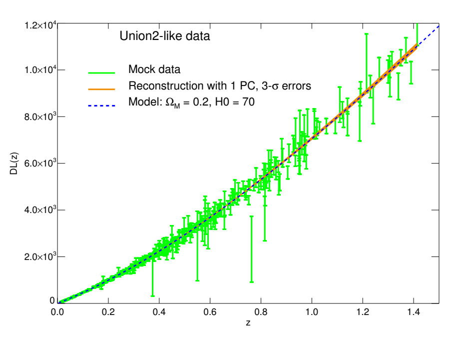
PCs for the Union2.1 catalog

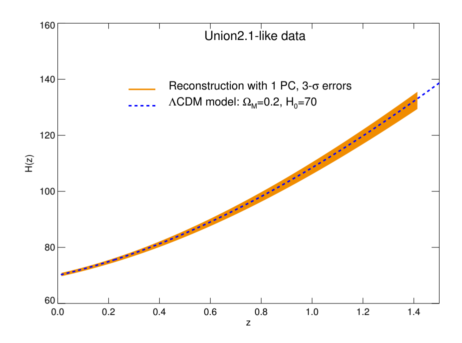




Tests with Mock Data

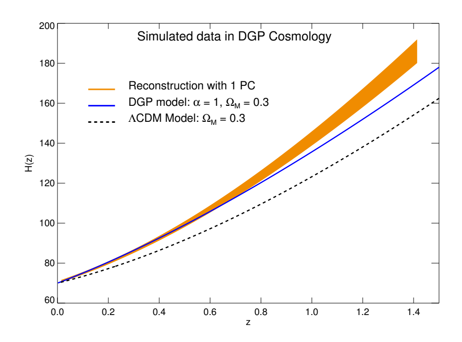
- ΛCDM: Several realizations varying Ω_m

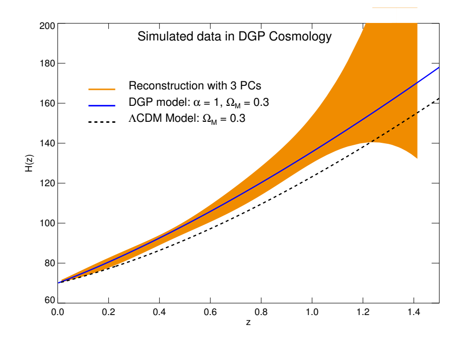




Tests with Mock Data

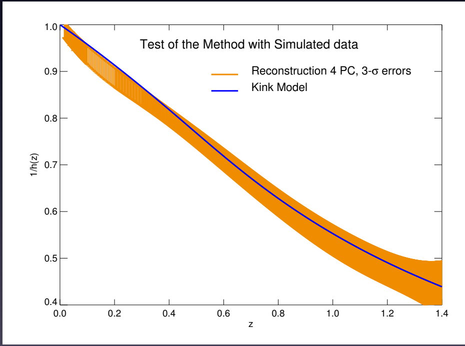
- Dvali-Gabadadze-Porrati (DGP) scenario
- IF higher number of PCs need to be calculated, indication that underlying cosmology might not be ΛCDM

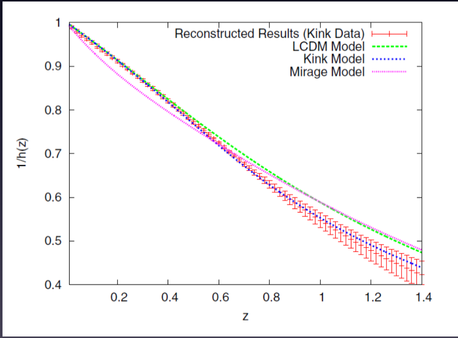




The Kink Model

- Extreme model with rapidly varying equation of state
- Reconstruction consistent with other techniques: Shafieloo & Kim 2011

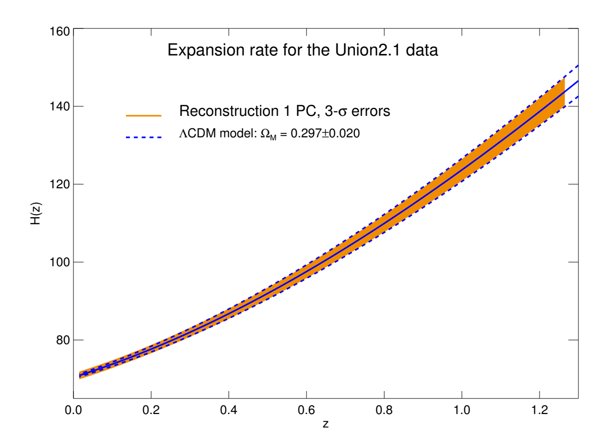




The Union2.1 Sample

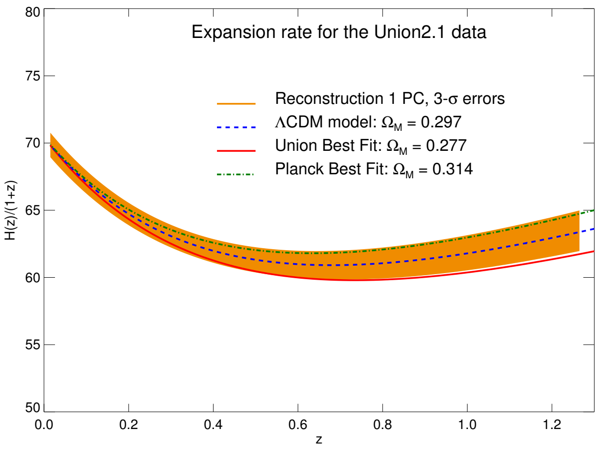
- From the behaviour of $H(z) \rightarrow \Omega_\mathrm{m} = 0.297 \pm 0.020$

$H(z)$

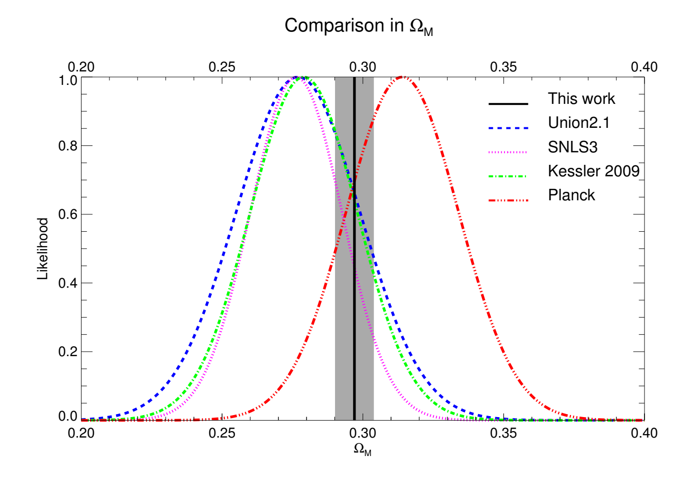


z

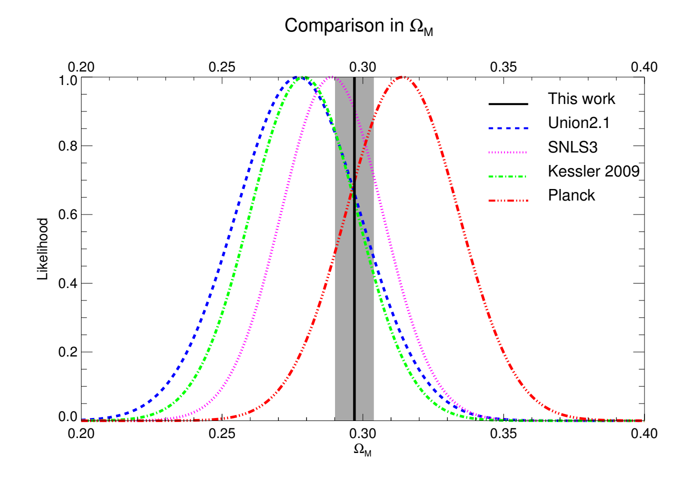
The Union2.1 Sample



Cosmological Constraints



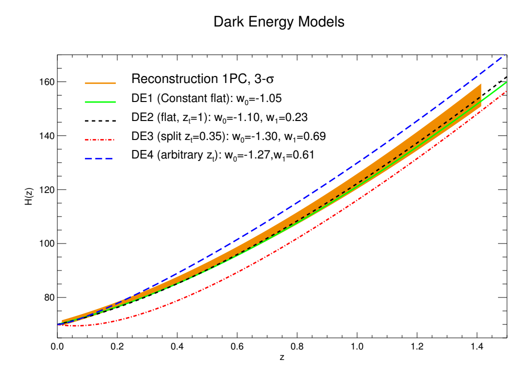
Cosmological Constraints



Other Cosmologies

1. Dark Energy models: extension of Chevallier+Polarski parameterization with varying z_t *(Rapetti et al. 2005)*

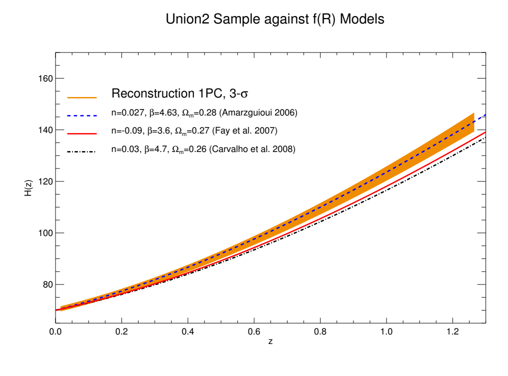
$$w(a) = \frac{w_{\rm et} z + w_0 z_{\rm t}}{z + z_{\rm t}} = \frac{w_{\rm et}(1-a)a_{\rm t} + w_0(1-a_{\rm t})a}{a(1-2a_{\rm t}) + a_{\rm t}}$$



Other Cosmologies

2. f(R) models *(Carvalho et al. 2008)*: $f(R) = R - \beta/R^n$

$$H^2 = H_0^2 \left[\frac{3\Omega_{m0}(1+z)^3 + f/H_0^2}{6f'\xi^2} \right]$$



Other Cosmologies

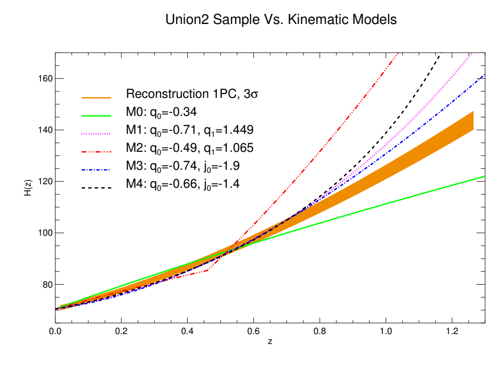
3. Kinematic models *(Guimaraes et al. 2009)*
Several parameterizations of the deceleration parameter $q(z)$

$$M0 : q(z) = q_0$$

$$M1 : q(z) = q_0 + q_1 z$$

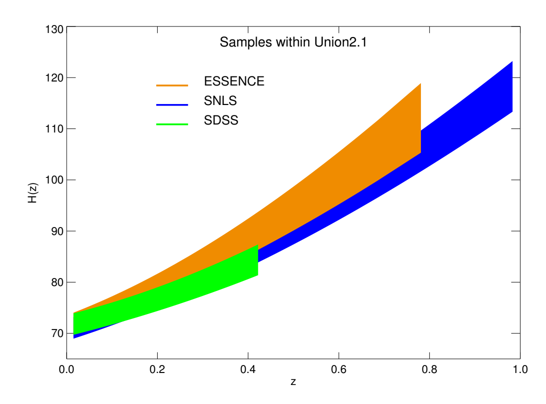
$$M2 : q(z) = q_0 \text{ for } z < z_t;$$
$$q(z) = q_1 \text{ for } z > z_t$$

$$M3 : j(z) = j_0$$



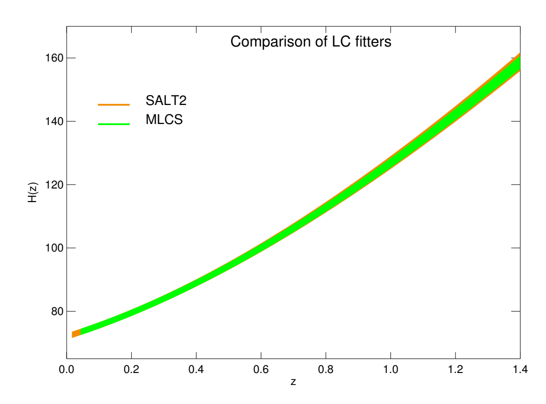
Systematic Errors

- Do different subsamples provide the same expansion history?



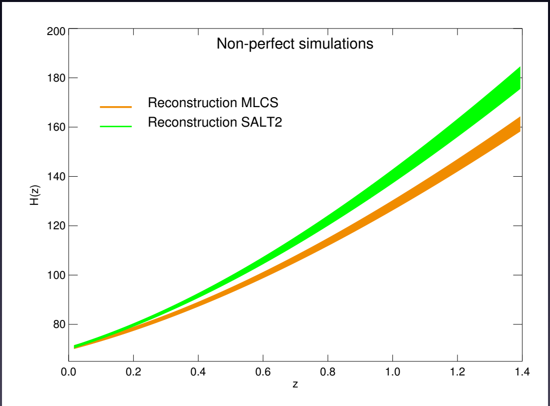
Systematic Errors

- Impact of light curve fitters in reconstructed $H(z)$
- Perfect simulations with SNANA



Systematic Errors

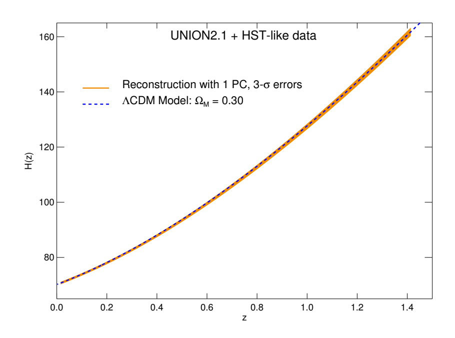
- Impact of light curve fitters in reconstructed $H(z)$
- Non-perfect simulations



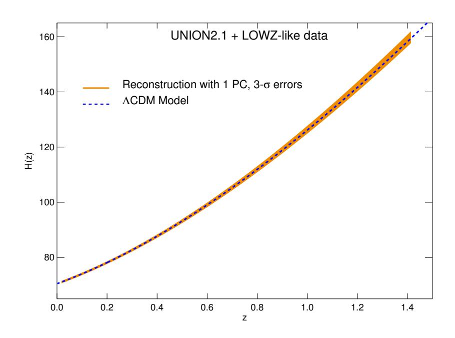
Testing Redshift Ranges

- Design a survey that is most sensitive to the expansion history in some specific redshift interval

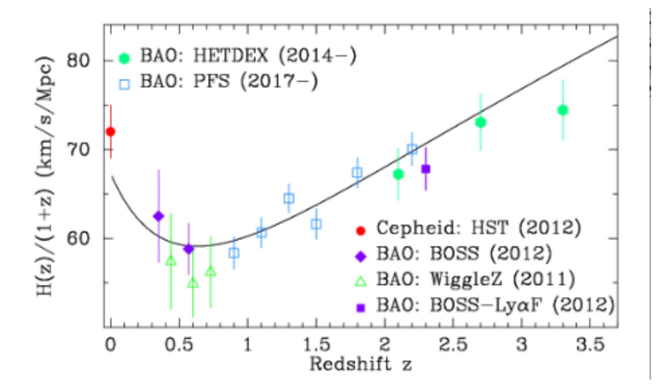
Union2.1 + High redshift



Union2.1 + Low redshift



Other Cosmological Probes



(Kim et al. 2013)

- Include BAOs: $D_{\mathrm{L}} = (1+z)^2 D_{\mathrm{A}}$

Summary

1) Model-independent approach to recover $H(z)$

2) Based on inversion of the distance-redshift relationship + PCA to build an *optimal basis* for the observable D_{L}

3) Framework to compare and constrain cosmological scenarios

4) Value for Ω_{m} agrees with new SNe constraints and Planck results

Looking ahead:

- BAO data and forecasts from future surveys, e.g. eBOSS
- Inhomogeneity: forecasts from LSST for ihnomogeneous models

THANK YOU

FOR YOUR ATTENTION!

Method Summary

$$\begin{bmatrix} D_{\rm L}(z_1, \Omega m_1, \Omega_{\Lambda_1}) - \bar{D}_{\rm L} \\ D_{\rm L}(z_2, \Omega m_2, \Omega_{\Lambda_2}) - \bar{D}_{\rm L} \\ D_{\rm L}(z_3, \Omega m_3, \Omega_{\Lambda_3}) - \bar{D}_{\rm L} \\ \cdots \\ D_{\rm L}(z_N, \Omega m_{\rm N}, \Omega_{\Lambda_{\rm N}}) - \bar{D}_{\rm L} \end{bmatrix}$$

Training Set

$$0.1 \leq \Omega_{\rm m} \leq 0.5$$

$$0.5 \leq \Omega_\Lambda \leq 0.9$$

PCA $\Rightarrow$ $\lambda_j, {\rm PC}_j$

SNe catalog

$\bar{D}_{\rm L} + \sum c_j {\rm PC}_j$

$$\chi^2 = \sum_i^{\rm SNe} \frac{[D_{\rm L}|_{{\rm obs},i} - D_{\rm L}|_{\rm mod}(a_i, c_j, H_0)]^2}{\sigma_i^2}$$

$$\Downarrow \quad \Downarrow$$

$$H_0, c_j$$

$$\Downarrow$$

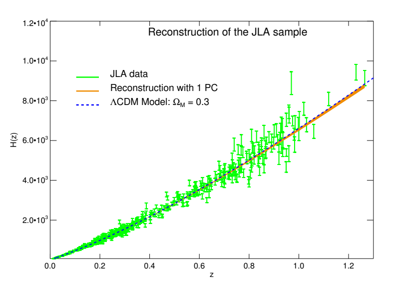
$$D_{\rm L}(a)$$

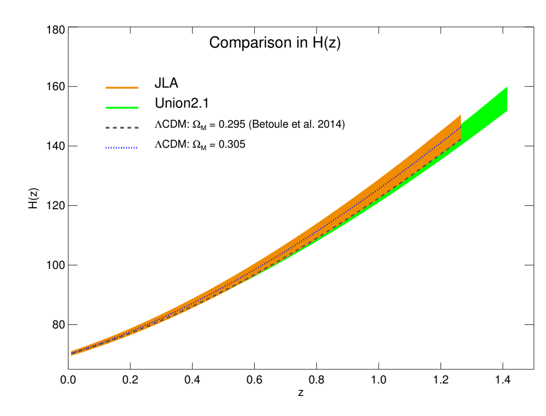
Neumann Series Machinery $\Downarrow$

$$\boxed{H(a)}$$

APPENDIX

The JLA sample





Uncertainties

- Fisher matrix of the χ^2 function

$$F_{ij} \equiv \left\langle \frac{\partial^2 \chi^2}{\partial c_i \partial c_j} \right\rangle \quad \rightarrow \quad F_{ij} = \sum_{k=0}^{\mathrm{SNe}} \frac{p_i(a_k) p_j(a_k)}{\sigma_k^2}$$

- Errors satisfy $(\Delta c_i)^2 = (F^{-1})_{ii}$ and propagate into estimate of $\Delta D_{\mathrm{L}}(a)$ and $\Delta e(a)$
- For D_{L}:

$$[\Delta D_{\mathrm{L}}(a)]^2 = \sum_{j=0}^{J} (\Delta c_j)^2 \, [p_j(a)]^2$$

- For $e(a)$:

$$[\Delta e(a)]^2 = \sum_{j=0}^{J} (\Delta c_j)^2 [e_n^j(a)]^2$$

- Error in determination of PCs: $\Delta\lambda = \frac{1}{\sqrt{\lambda}}$
- Total error budget

$$\boxed{\Delta_{\mathrm{T}}^2 = \Delta_{\mathrm{coeff}}^2 + \Delta_{H_0}^2 + \Delta_{\lambda}^2}$$

Equations

Hilbert-Einstein action

$$S = \frac{-1}{16\pi G} \int d^4 x R \sqrt{-g}$$

Friedmann Equation:

$$H^2(a) = H_0^2 \left[\frac{\Omega_{\mathrm{r}0}}{a^4} + \frac{\Omega_{\mathrm{m}0}}{a^3} - \frac{\Omega_{k0}}{a^2} + \Omega_{de0} F(a) \right]$$

$$F(a) = \exp \left[-3 \int_a^1 [1 + w(a')] \frac{da'}{a'} \right]$$

Equations II

Dark Energy

$$w(a) = \frac{w_{\mathrm{et}} z + w_0 z_{\mathrm{t}}}{z + z_{\mathrm{t}}} = \frac{w_{\mathrm{et}}(1-a)a_{\mathrm{t}} + w_0(1-a_{\mathrm{t}})a}{a(1-2a_{\mathrm{t}}) + a_{\mathrm{t}}}$$

DGP

$$H^2 - \frac{H^\alpha}{r_c^{2-\alpha}} = H_0^2 \left[\frac{\Omega_{\mathrm{m}}}{a^3}\right]; \quad r_c = (1-\Omega_{\mathrm{m}})^{\frac{1}{\alpha-2}} H_0^{-1}$$

$f(R)$

$$H^2 = H_0^2 \left[\frac{3\Omega_{\mathrm{m}0}(1+z)^3 + f/H_0^2}{6f'\xi^2}\right]$$

where ξ^2 depends on first and second derivatives of $f(R)$

KINEMATIC MODELS

$M0 : q(z) = q_0$

$M1 : q(z) = q_0 + q_1 z$

$M2 : q(z) = q_0$ for $z < z_t$; $q(z) = q_1$ for $z > z_t$

$M3 : j(z) = j_0$

Light Curve fitter models

SALT2

$$F(SN, p, \lambda) = x_0 \times [M_0(p, \lambda) + x_1 \times M_1(p, \lambda)] \times \exp[c \times CL(\lambda)]$$

$$\mu_B = m_B + \alpha x_1 - \beta c - M_B$$

MLCS

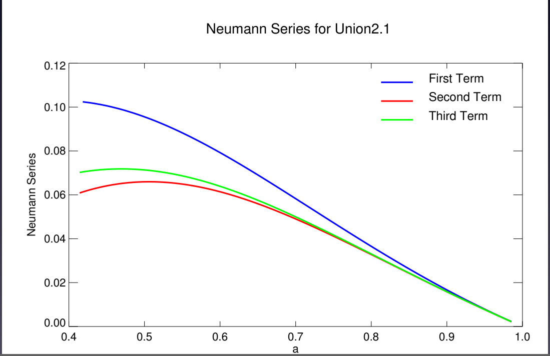
$$m^{e,f}_{\mathrm{model}} = M^{e,f'} + p^{e,f'}\Delta + q^{e,f'}\Delta^2 + X_{\mathrm{host}}{}^{e,f'} + K^{e}_{ff'} + \mu + X^{e,f'}_{\mathrm{MW}}$$

where $f \equiv$ observer frame filters; $f' \equiv$ rest-frame filters.
$M^{e,f'} \equiv$ absolute mag. for a SN with $\Delta = 0$

Neumann Series

$$e(a) = -a^3 D'_{\mathrm{L}} + a \int_1^{a_{\min}} x^{-2} e_{n-1}(x)dx$$

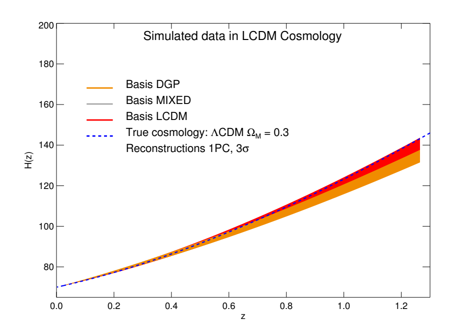
$$e(a) = \sum_{j=1}^{J} c_j e^{(j)}(a) = \sum_{j=1}^{J} c_j \left[-a^3 PC_j + \sum_{n=1}^{3} a^n \int_1^{a_{\min}} x^{-2} e_{n-1}(x)dx \right]$$



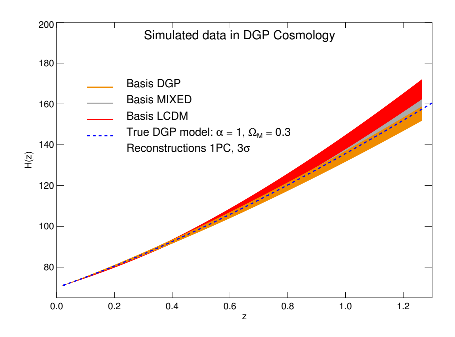
Extending the Training Set

- Training Set composed by ΛCDM behaviours only
- Make this initial matrix as general as possible
- Three different basis: ΛCDM, DGP and mixed

ΛCDM simulated data



DGP simulated data

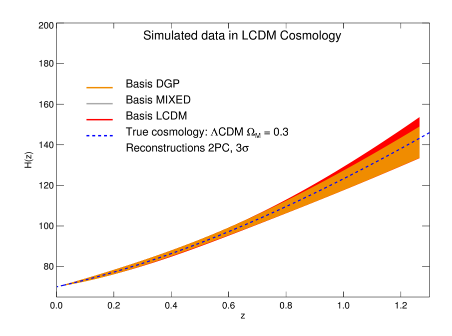


(Benitez-Herrera et al. in prep)

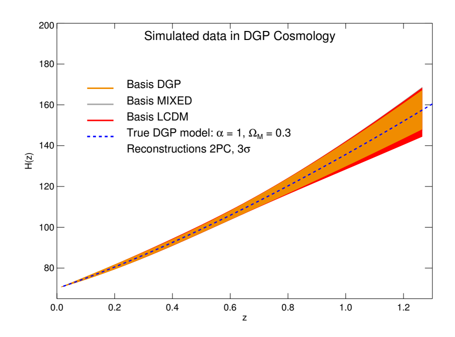
Extending the Training Set

- With 2PCs, the dependance with basis is negligible

ΛCDM simulated data



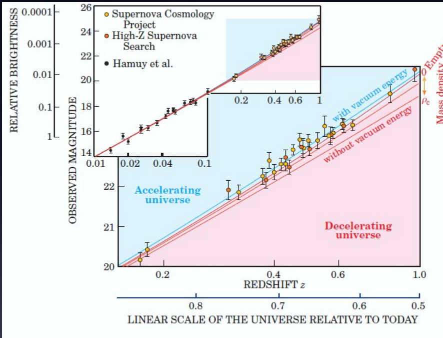
DGP simulated data



(Benitez-Herrera et al. in prep)

Cosmology with SNe Ia

- SNe beyond $z \sim 0.5$ appear fainter than expected for an empty universe
- First evidence of accelerated expansion
- Later confirmed by CMB and BAO measurements
- Nobel Prize in Physics 2011
  S. Perlmutter, B. Schmidt, A. Riess



(Perlmutter et al. 1999)

- For $D_{\rm L}$:

$$[\Delta D_{\rm L}(a)]^2 = \sum_{j=0}^{J} (\Delta c_j)^2 \left[p_j(a)\right]^2$$

- For $e(a)$:

$$[\Delta e(a)]^2 = \sum_{j=0}^{J} (\Delta c_j)^2 [e_n^j(a)]^2$$

Mocks

In 10 redshifts bins, z_i:

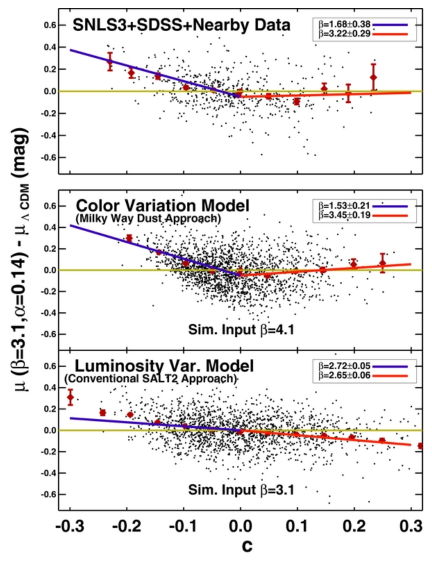
$DL = DL_{model} + e$

$e = m + \sigma * randomnumber$

with $\mu = 0, \sigma = 1$

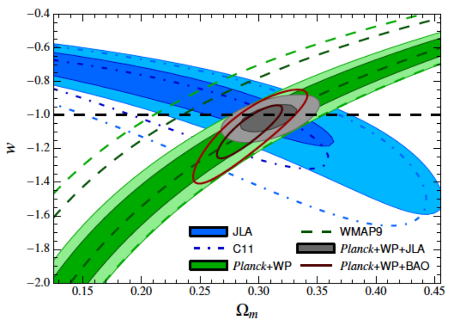
Systematic Errors

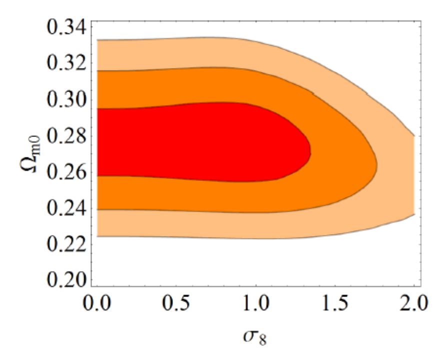
- SNANA-simulated samples with different systematics
- Test if detectable with PC algorithm



Current SN constraints

- Hubble diagram



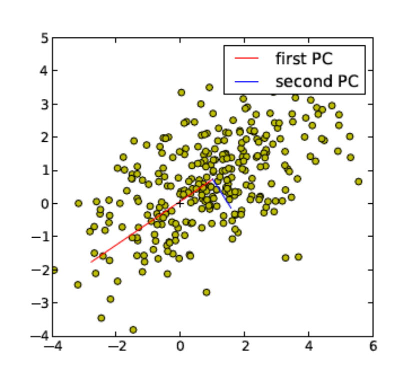


- $w = -1.027 \pm 0.055$
  (Betoule et al. 2014)

- $\sigma_8 = 0.84^{+0.28}_{-0.65}$
  (Castro & Quartin 2014)

Principal Component Analysis

- Statistical tool used in many astrophysical problems: e.g. reduction of data dimensionality



- Rotation of the parameter space: new variables are uncorrelated and orthogonal
- First components account for the largest variance in the data: directions of maximum clustering

Principal Component Analysis

- X_p^k where k is the number of data, p parameters
- Compute the covariance matrix (symmetric):

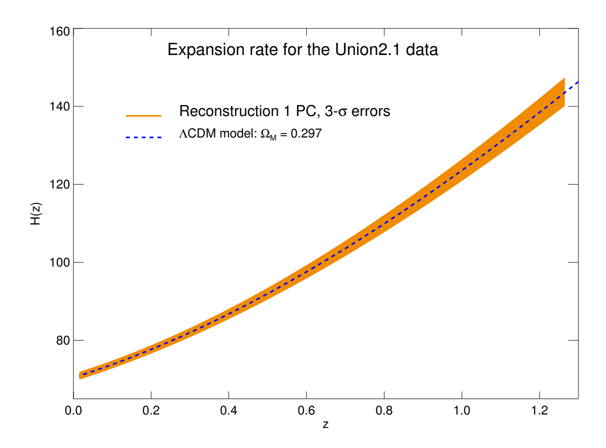
$$\Sigma_{ij} = \frac{\sum_k (X_i^k - \overline{X_i})(X_j^k - \overline{X_j})}{k}$$

- Diagonalize and sort the eigenvalues:

  $\lambda_1 > ... > \lambda_p \rightarrow w_1(\lambda_1), ..., w_p(\lambda_p)$
- Reproduce the initial data in a lower-dimensional space

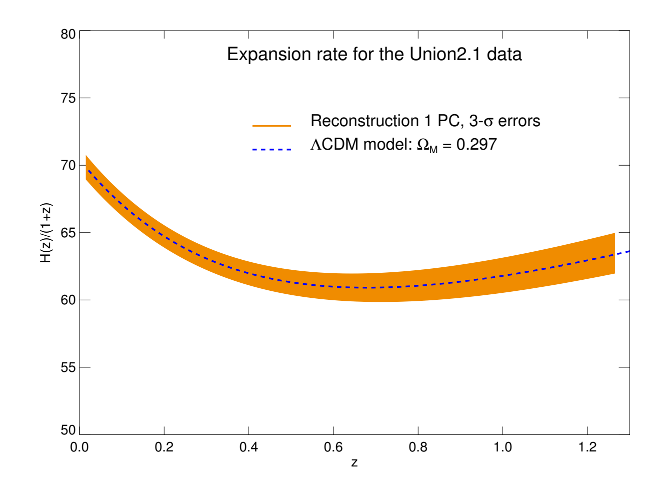
The Reconstructed Expansion Rate

$H(z)$



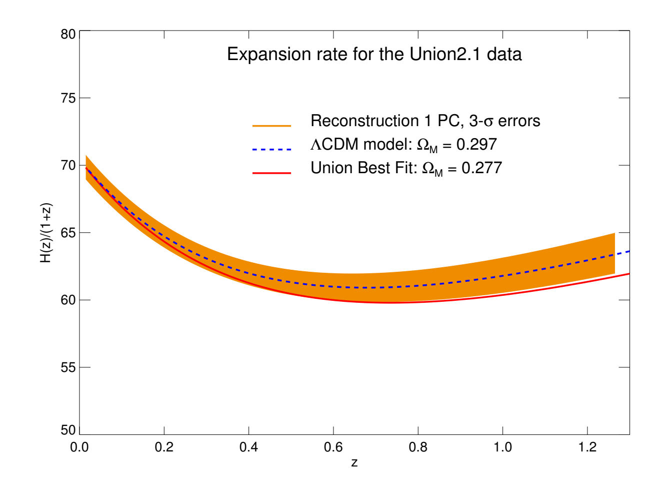
z

The Reconstructed Expansion Rate



(Benitez-Herrera et al. in prep)

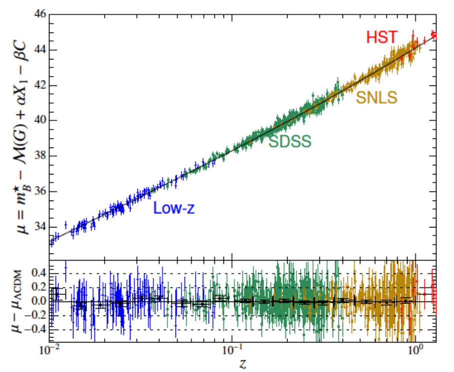
The Reconstructed Expansion Rate

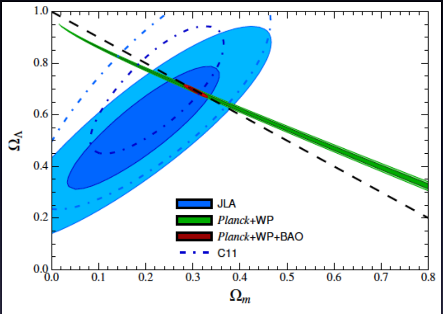


(Benitez-Herrera et al. in prep)

Current SN constraints

Hubble diagram: Intrinsic dispersion $\sigma_{\mathrm{m}} \sim 0.15$ mag



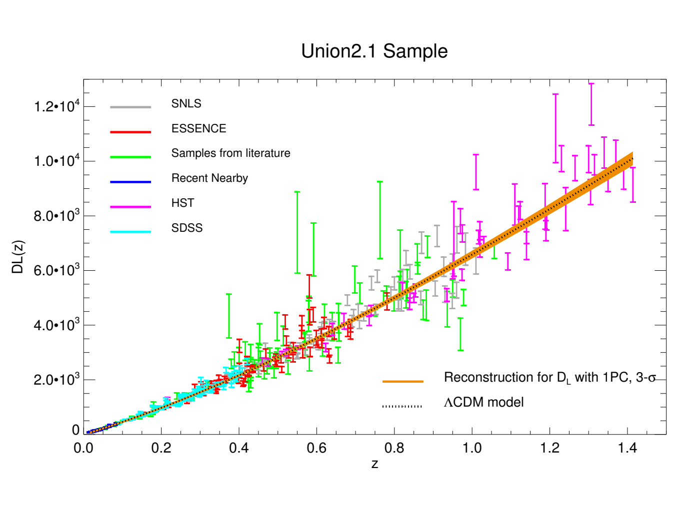


Consistent with a ΛCDM with $\Omega_{\mathrm{m}} = 0.295 \pm 0.034$

(Betoule et al. 2014)

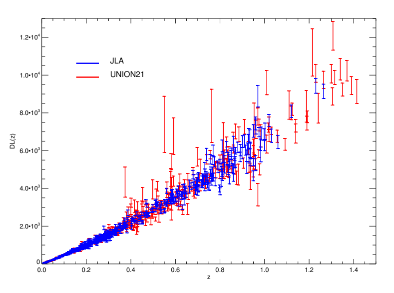
Union2.1 Sample *(Suzuki et al. 2012)*

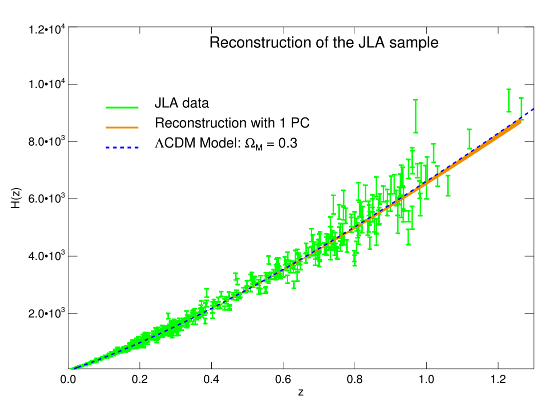
$D_{\rm L}(z)$



z *(Benitez-Herrera et al. 2013)*

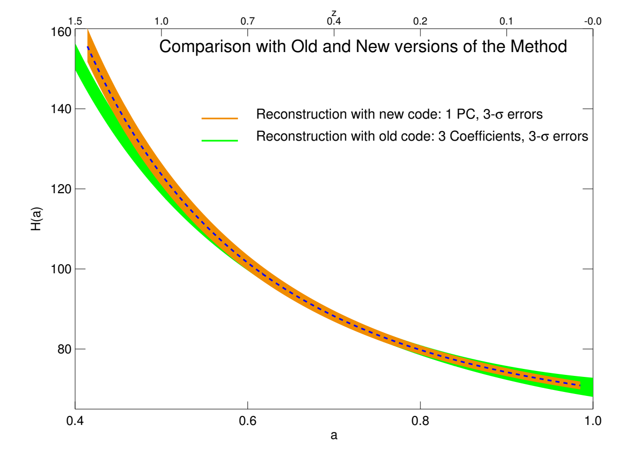
The JLA sample





The Reconstructed $H(a)$

$H(a)$



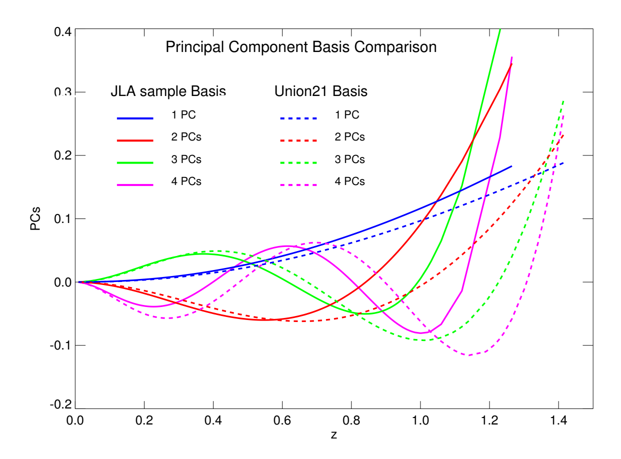
a

- Cumulative percentage of total variance

$$t_M = \left(\frac{\sum_{i=1}^{L} \lambda_i}{\sum_{j=1}^{N_{\rm PC}} \lambda_j} \right) \times 100$$

- Example: Union2.1 data
  1st PC accounts for 99.90% of t_M

PC basis for real data



Systematic Errors II

Source	Error on w
Vega	0.033
All instrument calibration	0.030
Color correction	0.020
Mass correction	0.016
Contamination	0.016
Intergalactic extinction	0.013
Galactic extinction	0.010
Light curve shape	0.006
Quadrature sum	**0.061**